RICEVIMENTO 9

Titolo nota 13/12/2007

$\boxed{\text{PAG 111}}$

$$\int_{-\infty}^{+\infty} \frac{1}{x^4+1}\,dx = \int_{-\infty}^{0} + \int_{0}^{+\infty}$$

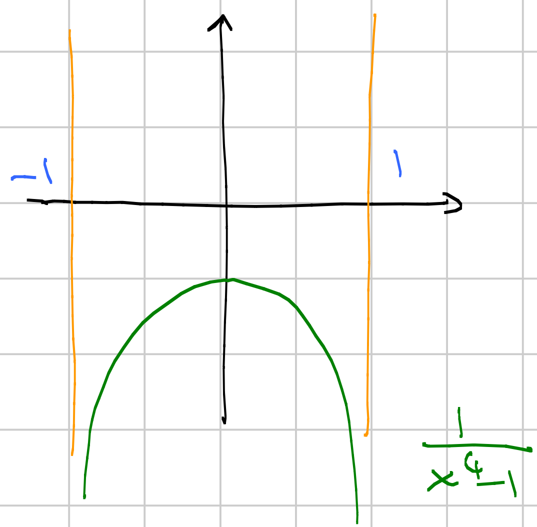
2 pezzi con stesso comportamento (la funzione è PARI)

$$\frac{1}{1+x^4} \sim \frac{1}{x^4} \quad \text{per } x \to +\infty \quad \Rightarrow \text{converge}$$

$\boxed{\text{PAG 111}}$

$$\int_{-1}^{1} \frac{1}{x^4-1}\,dx = \int_{-1}^{0} + \int_{0}^{1} = -\infty$$

stesso comportamento



Basta esaminare il problema in $x=1$

$$\frac{1}{x^4-1} = \frac{1}{(x-1)(x^3+x^2+x+1)}$$

COLPEVOLE

Idea: $\int \frac{1}{x^4-1}\,dx$ con pb. in $x=1$ si comporta come

$\int \frac{1}{x-1}\,dx$ con pb. in $x=1$

Con il cambio di variabile si riporta il pb. in 0 e si vede che diverge.

Formalmente: C.A. con $g(x) = \frac{1}{x-1}$

$$\lim_{x \to 1^-} \frac{\frac{1}{x^4-1}}{\frac{1}{x-1}} = \lim_{x \to 1^-} \frac{1}{\cancel{(x-1)}(x^3+x^2+x+1)} \cdot \cancel{(x-1)} = \frac{1}{4}$$

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$\boxed{\text{PAG 113}}$

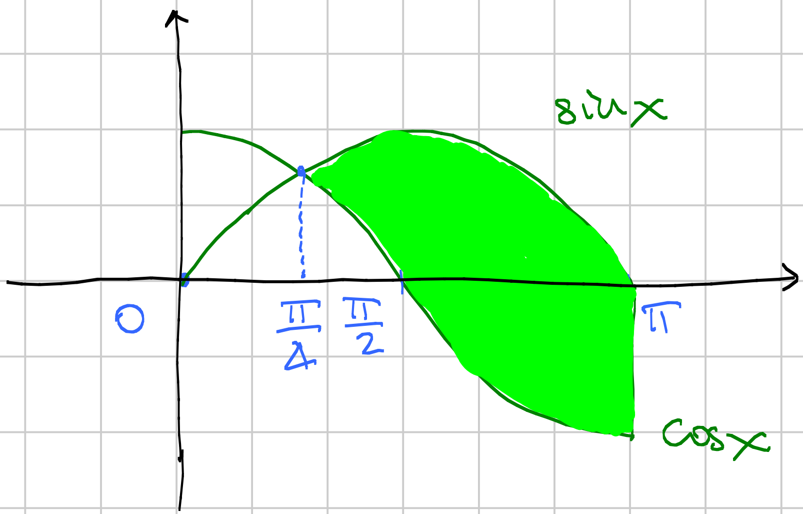
$$\{(x,y) \in \mathbb{R}^2 : 0 \le x \le \pi,\ \cos x \le y \le \sin x\} = A$$

Calcolare x_a e y_a

$$\text{Area} = \int_{\frac{\pi}{4}}^{\pi} (\sin x - \cos x)\,dx$$

$$= \Big[-\cos x - \sin x\Big]_{x=\frac{\pi}{4}}^{x=\pi}$$

$$= 1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = 1 + \sqrt{2}$$



$$\iint_A x\,dx\,dy = \int_{\frac{\pi}{4}}^{\pi} dx \int_{\cos x}^{\sin x} x\,dy = \int_{\frac{\pi}{4}}^{\pi} x\,dx \int_{\cos x}^{\sin x} dy =$$

$$= \int_{\frac{\pi}{4}}^{\pi} (x\sin x - x\cos x)\,dx = \Big[-x\cos x + \sin x - x\sin x - \cos x\Big]_{x=\frac{\pi}{4}}^{x=\pi}$$

$$= \pi + 1 + \frac{\pi}{4}\frac{\sqrt{2}}{2} - \cancel{\frac{\sqrt{2}}{2}} + \frac{\pi}{4}\frac{\sqrt{2}}{2} + \cancel{\frac{\sqrt{2}}{2}}$$

$$= \pi + 1 + \frac{\pi\sqrt{2}}{4}$$

$$x_G = \frac{1}{\text{Area}} \iint x = \frac{1}{1+\sqrt{2}} \left(\pi + 1 + \pi \frac{\sqrt{2}}{4} \right)$$

$$\iint_A y \, dx \, dy = \int_{\frac{\pi}{4}}^{\pi} dx \int_{\cos x}^{\sin x} y \, dy = \int_{\frac{\pi}{4}}^{\pi} dx \left[\frac{y^2}{2} \right]_{y=\cos x}^{y=\sin x} =$$

$$= \frac{1}{2} \int_{\frac{\pi}{4}}^{\pi} (\sin^2 x - \cos^2 x) \, dx = -\frac{1}{2} \int_{\frac{\pi}{4}}^{\pi} \cos(2x) \, dx$$

$$= -\frac{1}{2} \left[\frac{\sin(2x)}{2} \right]_{x=\frac{\pi}{4}}^{x=\pi}$$

$$= -\frac{1}{2} \left(-\frac{1}{2} \right) = \frac{1}{4}$$

$$y_G = \frac{1}{4 \text{ Area}} = \frac{1}{4 + 4\sqrt{2}}$$

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$$A = \{ (x,y) \in \mathbb{R}^2 : 1 \le x^2+y^2 \le 2x \}$$

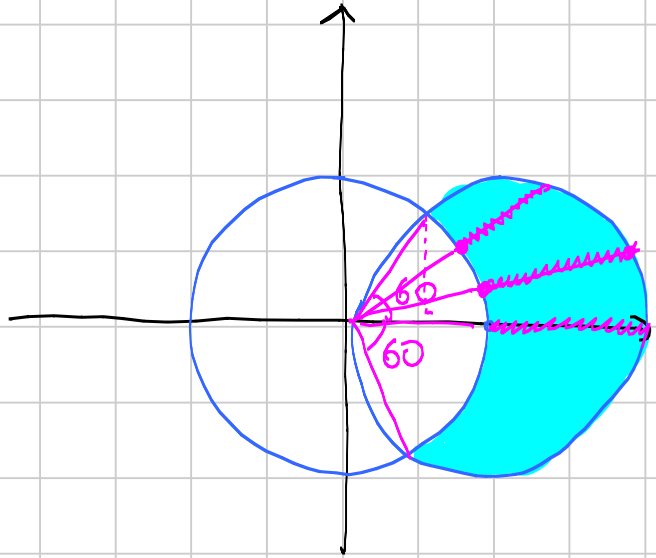
$1 \le x^2+y^2 \to$ Fuori dal cerchio con centro in $(0,0)$ e raggio 1

$x^2+y^2 \le 2x$

$x^2-2x+y^2 \le 0$

$x^2-2x+1+y^2 \le 1$

$(x-1)^2+y^2 \le 1 \to$ Dentro il cerchio con centro in $(1,0)$ e raggio 1



$x^2+y^2 \ge 1$ diventa $\boxed{\rho \ge 1}$; $x^2+y^2 \le 2x$ diventa $\rho^2 \le 2\rho\cos\theta$, cioè

$\boxed{\rho \le 2\cos\theta}$

In coord. polari A diventa

$$\theta \in \left[-\frac{\pi}{3}, \frac{\pi}{3}\right], \qquad 1 \le \rho \le 2\cos\theta$$

$$\text{Area}(A) = \int_{-\frac{\pi}{3}}^{\pi/3} d\theta \int_1^{2\cos\theta} d\rho \; 1 \cdot \rho \quad = \ldots$$

$$\iint_A x \, dx\, dy = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} d\theta \int_1^{2\cos\theta} d\rho \; \underbrace{\rho\cos\theta}_{x} \cdot \underbrace{\rho}_{J}$$

$$\iint_A y \, dx\, dy = 0 \quad \text{per simmetria.}$$

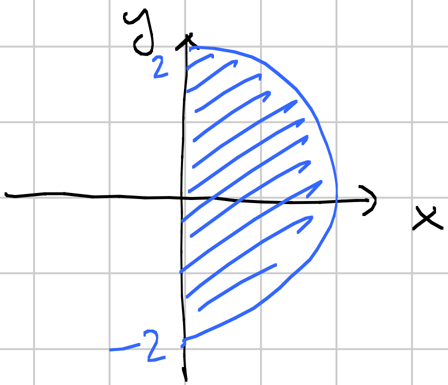
$\boxed{\text{Pag. 107}}$

$$\iiint_A z\sqrt{x^2+y^2}\,dx\,dy\,dz$$

$$A = \{(x,y,z)\in\mathbb{R}^2 : \underbrace{x^2+y^2\le 4,\ z\in[0,3]}_{\text{CILINDRO}},\ \underbrace{x\ge 0}_{\text{METÀ CILINDRO}}\}$$

In coordinate cilindriche:

$$z\in[0,3],\quad \rho\in[0,2],\quad \theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$



$$\iiint_A z\sqrt{x^2+y^2}\,dx\,dy\,dz =$$

$$\int_0^3 dz\int_0^2 d\rho\int_{-\frac{\pi}{2}}^{\pi/2} d\theta\ \ z\underbrace{\rho}_{\sqrt{x^2+y^2}}\cdot\overset{J}{\rho} = \underbrace{\int_0^3 z\,dz}_{\frac{9}{2}}\ \underbrace{\int_0^2\rho^2\,d\rho}_{\frac{8}{3}}\ \underbrace{\int_{-\frac{\pi}{2}}^{\pi/2} d\theta}_{\pi} = 12\pi.$$

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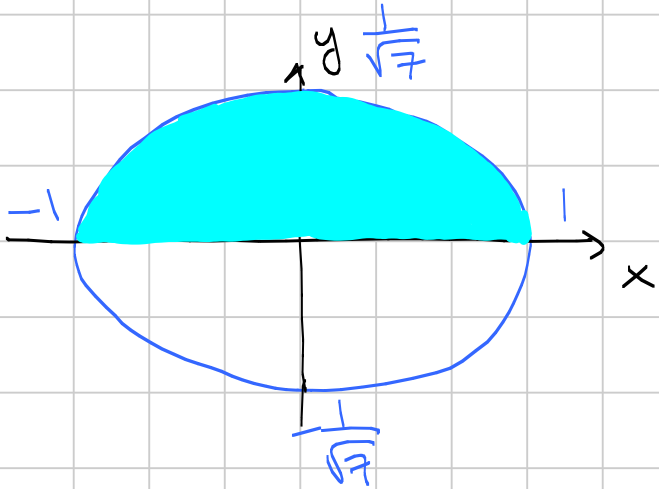
$$\iint_A \sin x \, dx\, dy = 0 \qquad A = \{ (x,y) \in \mathbb{R}^2 : x^2+7y^2 \le 1,\ y \ge 0 \}$$



$\sin x$ è dispari nella variabile x
L'insieme è simmetrico rispetto all'asse y

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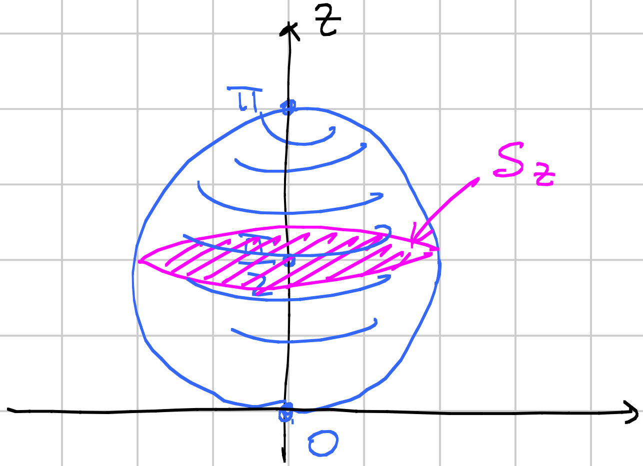
$$A = \{ (x,y,z) \in \mathbb{R}^3 : z \in [0,\pi],\ \underbrace{x^2+y^2 \le \sin z}_{\text{Descrizione di } S_z} \}$$

$$\iiint_A z^2 \, dx\, dy\, dz$$

Fissato $z \in [0,\pi]$, (x,y) varia in un cerchio di raggio $\sqrt{\sin z}$



$$\iiint_A z^2\,dx\,dy\,dz \overset{\text{SEZIONI}}{=} \int_0^3 dz \iint_{S_z} z^2\,dx\,dy$$

$$= \int_0^3 z^2\,dz \iint_{S_z} dx\,dy = \int_0^3 z^2 \text{ area } (S_z)\,dz = \pi \int_0^3 z^2 R_z^2\,dz$$

$$= \pi \int_0^3 z^2 \sin z\,dz = \text{per parti}\ldots$$

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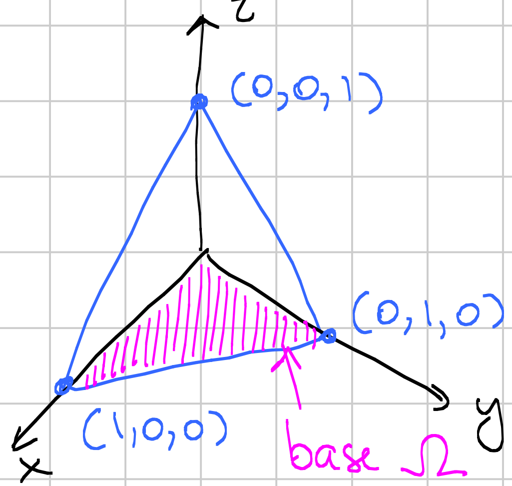
$\boxed{\text{Pag } 107}$

$$A = \{(x,y,z) \in \mathbb{R}^3 : x \geq 0,\ y \geq 0,\ z \geq 0,\ x+y+z \leq 1\}$$

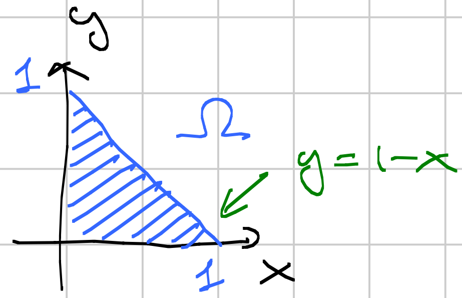
$x+y+z \leq 1 \iff z \leq 1-x-y$

"sotto il piano di equazione $x+y+z=1$"



$$A = \{ (x,y,z) \in \mathbb{R}^3 : \underbrace{(x,y)\in\Omega,}_{\text{BASE}} \ \underbrace{0 \le z \le 1-x-y}_{\text{COLONNA}} \}$$

$$\iiint_A x\,dx\,dy\,dz \overset{\text{COLONNE}}{=} \iint_\Omega dx\,dy \int_0^{1-x-y} dz \quad x = \iint_\Omega x\,dx\,dy \int_0^{1-x-y} dz$$



$$= \iint_\Omega x\,(1-x-y)\,dx\,dy$$

$$= \int_0^1 dx \int_0^{1-x} dy \ x\,(1-x-y)$$

$$= \int_0^1 x\,dx \int_0^{1-x} (1-x-y)\,dy = \int_0^1 x \left[y - xy - \frac{y^2}{2} \right]_{y=0}^{y=1-x}$$

= si fa...

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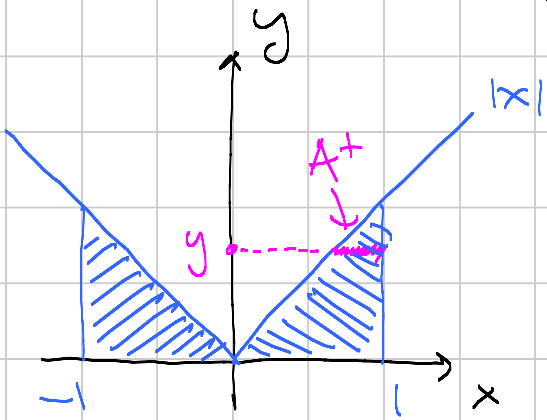
$$\iint_A |x| \arctan y \, dx\, dy = \qquad A = \{(x,y) \in \mathbb{R}^2 : x \in [-1,1],\ 0 \le y \le |x|\}$$



$$= 2 \iint_{A^+} x \arctan y \, dx\, dy$$

$$= 2 \int_0^1 dx \int_0^x dy \; x \arctan y$$

Normale asse x

$$= 2 \int_0^1 x\, dx \int_0^x \arctan y \, dy = \text{provare} \ldots$$

$$\iint_{A^+} x \arctan y \, dx\, dy =$$

Normale asse y

$$A^+ = \{(x,y) \in \mathbb{R}^2 : y \in [0,1],\ y \le x \le 1\}$$

$$= \int_0^1 dy \int_y^1 x \arctan y \, dx =$$

$$= \int_0^1 \arctan y \, dy \int_y^1 x \, dx = \int_0^1 \arctan y \left[\frac{x^2}{2} \right]_{x=y}^{x=1} dy$$

$$= \frac{1}{2} \int_0^1 \underbrace{\arctan y}_{G} \cdot \underbrace{(1-y^2)}_{f} \, dy = \text{per parti}$$

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