RICEVIMENTO 7

Titolo nota 29/11/2007

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$$\iint_A e^{3x} \cos(x+y)\,dx\,dy \qquad A = [0,\pi]\times[0,\pi]$$

$$= \int_0^\pi dx \int_0^\pi e^{3x} \cos(x+y)\,dy = \int_0^\pi e^{3x}\,dx \int_0^\pi \cos(x+y)\,dy =$$

$$= \int_0^\pi e^{3x}\,dx \,\Big[+\sin(x+y)\Big]_{y=0}^{y=\pi} = \int_0^\pi e^{3x}\,dx \,\{+\sin(x+\pi) - \sin x\}$$

$$= -2\int_0^\pi \sin x \cdot e^{3x}\,dx = -2\left[\frac{1}{10}\left[-\cos x + 3\sin x\right]e^{3x}\right]_{x=0}^{x=\pi} = -\frac{1}{5}\left(e^{3\pi}+1\right)$$

$$\int \underset{f}{\sin x} \cdot \underset{G}{e^{3x}}\,dx = \underset{F}{-\cos x} \cdot \underset{G}{e^{3x}} - \int \underset{F}{(-\cos x)} \cdot \underset{g}{3e^{3x}}\,dx$$

$$= -\cos x \cdot e^{3x} + 3\int \underset{f}{\cos x}\, \underset{G}{e^{3x}}\,dx$$

$$= -\cos x \cdot e^{3x} + 3\left[\underset{F}{\sin x}\cdot \underset{G}{e^{3x}} - \int \underset{F}{\sin x}\cdot \underset{g}{3e^{3x}}\,dx\right]$$

$$\int \sin x \cdot e^{3x}\,dx = -\cos x \cdot e^{3x} + 3\sin x \cdot e^{3x} - 9\int \sin x \cdot e^{3x}\,dx$$

grande ritorno

$$\int \sin x \cdot e^{3x}\,dx = \frac{1}{10}\left[-\cos x + 3\sin x\right]e^{3x}$$

— o — o —

$$\sum_{n=1}^{\infty} \underbrace{\frac{n^2 \log n + 2}{n^4 + 4}}_{a_n}$$

Brutale! $a_n \sim \frac{n^2 \log n}{n^4} = \frac{\log n}{n^2}$

$\sum \frac{\log n}{n^2}$ C.A. con $\frac{1}{n^{3/2}}$ caso limite...

$\to$ converge

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$$\sum_{n=1}^{\infty} \overbrace{\frac{\alpha^n}{(n^{2\alpha}+1)(n+|\alpha|)}}^{a_n} \qquad \alpha \geq 0 \qquad \frac{\alpha^n}{n^{2\alpha+1}}$$

$\alpha > 1$ $\quad \dfrac{\alpha^n}{\sim} \rightarrow +\infty \quad \dfrac{\text{esponenziale}}{\text{potenze}}$

$\Rightarrow$ no condiz. nec. $\Rightarrow$ non converge

$\alpha = 1$ $\quad \dfrac{1}{(n^2+1)(n+1)} \Rightarrow$ converge $\sim \dfrac{1}{n^3}$

$0 \leq \alpha < 1$ $\quad$ Radice $\sqrt[n]{a_n} = \dfrac{\alpha}{\sqrt[n]{\text{potenze}}} \rightarrow \alpha < 1$

$\Rightarrow$ per il criterio della radice converge.

VOLENDO per $\alpha \neq 1$ funziona sempre il criterio della radice

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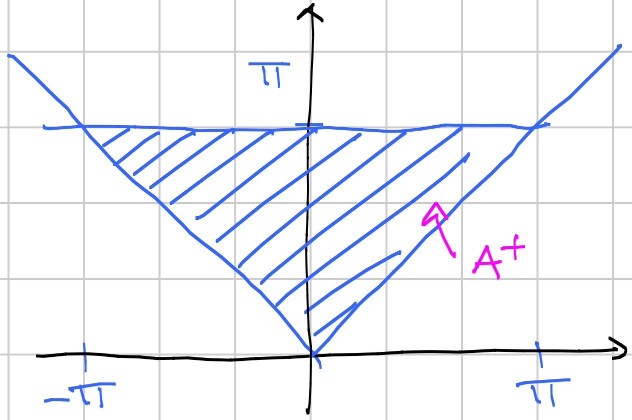
$$\iint_A |x \sin y| \, dx\, dy \qquad A = \{(x,y) \in \mathbb{R}^2 : |x| \le y \le \pi\}$$



$$= 2 \iint_{A^+} x \sin y \, dx\, dy =$$

$$= 2 \int_0^{\pi} dx \int_x^{\pi} dy \; x \sin y$$

$$= 2 \int_0^{\pi} x\, dx \int_x^{\pi} \sin y \, dy = 2 \int_0^{\pi} x \, dx \, \Big[-\cos y\Big]_{y=x}^{y=\pi} =$$

$$= 2 \int_0^{\pi} x\, dx \, (1 + \cos x) = 2 \int_0^{\pi} x \, dx + 2 \int_0^{\pi} \underset{G}{x} \, \underset{f}{\cos x} \, dx =$$

$$= \Big[x^2\Big]_{x=0}^{x=\pi} + 2 \left(+ \underset{G}{x} \, \underset{F}{\sin x} - \int_0^{\pi} \sin x \, dx \right) =$$

$$= \pi^2 + 2 \Big[x \sin x\Big]_{x=0}^{x=\pi} - 2 \Big[-\cos x\Big]_{x=0}^{x=\pi} = \boxed{\pi^2 - 4}$$

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$$\iint_A x \sin(xy)\, dx\, dy \qquad [1,2]\times[0,\pi] = A$$

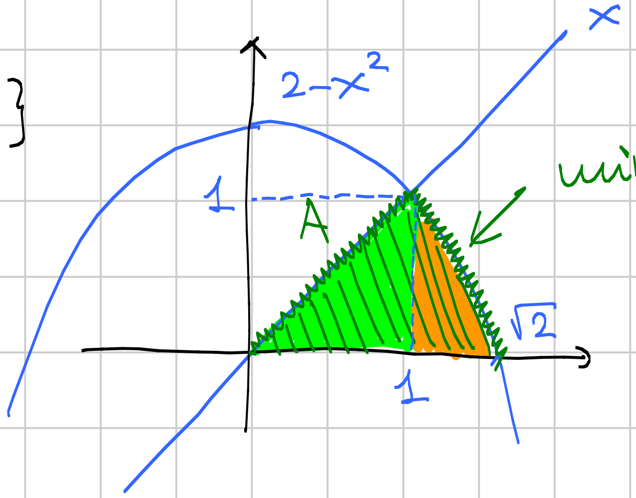
$$\int_1^2 dx \int_0^{\pi} x \sin(xy)\, dy = \int_1^2 dx \left[-\cos(xy)\right]_{y=0}^{y=\pi} =$$

$$= \int_1^2 \left(-\cos(\pi x)+1\right) dx = \left[-\frac{\sin(\pi x)}{\pi} + x\right]_{x=1}^{x=2} = 1.$$

$\boxed{\text{PAG } 105}$

$$\iint_A (x-1)\, dx\, dy \qquad A = \{(x,y)\in\mathbb{R}^2 : x\in[0,\sqrt{2}],\ 0\le y\le \min\{x, 2-x^2\}\}$$

$\min\{x, 2-x^2\}$



$$\iint_A (x-1)\,dx\,dy = \int_0^1 dx \int_0^x (x-1)\,dy + \int_1^{\sqrt{2}} dx \int_0^{2-x^2} (x-1)\,dy$$

$$= \int_0^1 (x-1)\,dx \int_0^x dy + \int_1^{\sqrt{2}} (x-1)\,dx \int_0^{2-x^2} dy$$

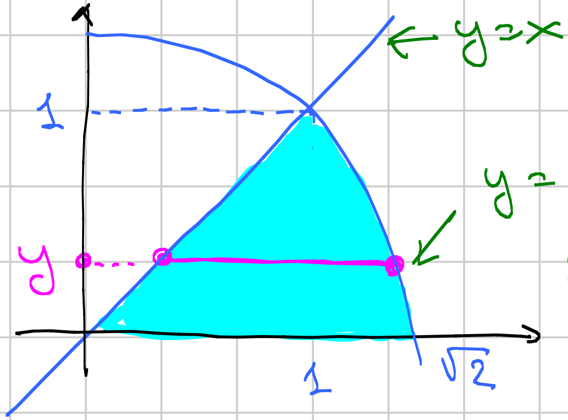
$$= \int_0^1 (x-1)\,x\,dx + \int_1^{\sqrt{2}} (x-1)(2-x^2)\,dx = \dots$$

Normale rispetto asse y

$$A = \{(x,y) \in \mathbb{R}^2 : \ y \in [0,1], \quad y \le x \le \sqrt{2-y}\}$$

$$\iint_A (x-1)\,dx\,dy = \int_0^1 dy \int_y^{\sqrt{2-y}} (x-1)\,dx =$$

$$= \int_0^1 dy \left[\frac{x^2}{2} - x \right]_{x=y}^{x=\sqrt{2-y}} = \dots$$



$x^2 = 2-y$

$x = \sqrt{2-y}$

$$A = \{ (x,y) \in \mathbb{R}^2 : y \in [0,1], \; y \le x \le \sqrt{2-y} \}$$

$\boxed{\text{PAG 104}}$

$$\int_2^4 \frac{1}{x^2-1}\,dx$$

si Ricavano A e B

$$\frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} = -\frac{1}{2}\frac{1}{x+1} + \frac{1}{2}\frac{1}{x-1}$$

$$\int \frac{dx}{x^2-1} = -\frac{1}{2}\int \frac{1}{x+1}\,dx + \frac{1}{2}\int \frac{1}{x-1}\,dx$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{2}\log|x-1|$$

$$= \frac{1}{2}\log\frac{|x-1|}{|x+1|}$$

$$\int_2^4 \frac{dx}{x^2-1} = \left[\frac{1}{2}\log\frac{|x-1|}{|x+1|}\right]_{x=2}^{x=4} = \frac{1}{2}\log\frac{3}{5} - \frac{1}{2}\log\frac{1}{3} =$$

$$= \frac{1}{2}\left(\log\frac{3}{5} - \log\frac{1}{3}\right) = \frac{1}{2}\log\left(\frac{3}{5}\cdot 3\right) = \frac{1}{2}\log\frac{9}{5} =$$

$$= \log\sqrt{\frac{9}{5}} = \log\left(\frac{3}{\sqrt{5}}\right)$$

$$\int \frac{1}{x^2-1}\,dx = \int \frac{1+x-x}{x^2-1}\,dx$$

$$= \int \frac{1+x-x}{(1+x)(x-1)}\,dx = \int \left[\frac{1}{x-1} - \frac{1}{2}\frac{2x}{x^2-1} \right] dx$$

$$= \log|x-1| - \frac{1}{2}\log|x^2-1|$$

$$= \log|x-1| - \log\sqrt{|x^2-1|}$$

$$= \log\frac{|x-1|}{\sqrt{|x^2-1|}} = \log\frac{|x-1|}{|x-1|^{1/2}\,|x+1|^{1/2}}$$

$$= \log\frac{|x-1|^{1/2}}{|x+1|^{1/2}} = \frac{1}{2}\log\frac{|x-1|}{|x+1|}$$