RICEVIMENTO 2

Titolo nota 18/10/2007

$$\left\{\log(1+n^2) - \textcircled{2}\log n^{2}\right\} n^2 \arctan n!$$

$$\log\left(\frac{1+n^2}{n^2}\right)\cdot n^2\cdot \arctan n! = \boxed{\frac{\log\left(1+\frac{1}{n^2}\right)}{\frac{1}{n^2}}}\cdot \underset{\downarrow\ \frac{\pi}{2}}{\boxed{\arctan n!}} \to \frac{\pi}{2}$$

$$\downarrow \quad x = \frac{1}{n^2}$$

$$\lim_{x\to 0}\frac{\log(1+x)}{x} = 1$$

$$\frac{2^n}{4^{\sqrt{n}}} = \frac{e^{n\log 2}}{e^{\sqrt{n}\log 4}} = e^{n\log 2-\sqrt{n}\log 4} \longrightarrow \text{"}e^{+\infty}\text{"} = +\infty$$

$$n\log 2 - \sqrt{n}\log 4 = \underset{\downarrow\ +\infty}{n}\left(\log 2 - \underset{\downarrow\ 0}{\boxed{\frac{\log 4}{\sqrt{n}}}}\right) \longrightarrow +\infty$$

$$\sum_{k=n}^{2n} \frac{1}{k^2} = \underbrace{\frac{1}{n^2} + \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \dots + \frac{1}{(2n)^2}}_{a_n}$$

$$\frac{1}{n^2} + \frac{1}{n^2} + \frac{1}{n^2} + \dots + \frac{1}{n^2} = \frac{n+1}{n^2}$$

$$\underset{\substack{\downarrow \\ 0}}{\boxed{0}} \le \underset{\substack{\rightsquigarrow \\ 0}}{\boxed{a_n}} \le \underset{\substack{\downarrow \\ 0}}{\boxed{\frac{n+1}{n^2}}}$$

— o — o —

$$\underset{\substack{\rightsquigarrow \\ +\infty}}{\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}} + \frac{1}{\sqrt{n+2}} + \dots + \frac{1}{\sqrt{2n}}} \ge \frac{1}{\sqrt{2n}} + \frac{1}{\sqrt{2n}} + \dots + \frac{1}{\sqrt{2n}}$$

$$= \underset{\substack{\downarrow \\ +\infty}}{\boxed{\frac{n+1}{\sqrt{2n}}}}$$

$$\frac{n!\,2^n}{n^n} = a_n$$ Rapporto

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)!\,2^{n+1}}{(n+1)^{(n+1)}}\ \frac{n^n}{n!\,2^n} = \frac{\cancel{(n+1)}\,\cancel{n!}\,2\cdot\cancel{2^n}}{\cancel{(n+1)}\,(n+1)^n}\ \frac{n^n}{\cancel{n!}\,\cancel{2^n}}$$

$$= 2\,\frac{n^n}{(n+1)^n} = \frac{2}{\left(\frac{n+1}{n}\right)^n} = \frac{2}{\left(1+\frac{1}{n}\right)^n} \longrightarrow \frac{2}{e} < 1$$

$$\Longrightarrow a_n \longrightarrow 0$$

— o — o —

$$n^3 \sin\frac{1}{n^3} = \frac{\sin\frac{1}{n^3}}{\frac{1}{n^3}} \longrightarrow 1$$

$$a_n = \frac{n!\,(4n)!}{(2n)!\,(3n)!}$$

Rapporto

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{(n+1)!\,(4n+4)!}{(2n+2)!\,(3n+3)!}}{\dfrac{n!\,(4n)!}{(2n)!\,(3n)!}}$$

$$= \frac{(n+1)\cancel{n!}\,(4n+4)(4n+3)(4n+2)(4n+1)\cancel{(4n)!}}{(2n+2)(2n+1)\cancel{(2n)!}\,(3n+3)(3n+2)(3n+1)\cancel{(3n)!}} \cdot \frac{\cancel{(2n)!}\,\cancel{(3n)!}}{\cancel{n!}\,\cancel{(4n)!}}$$

$$\to \frac{4^4}{4\cdot 27} = \frac{4^3}{27} = \frac{64}{27}$$

Rapporto → Radice

$$\frac{a_{n+1}}{a_n} \to \frac{64}{27} > 1 \Rightarrow a_n \to +\infty \quad ; \quad \sqrt[n]{a_n} \to \frac{64}{27}$$

$$\frac{\sin(8x^2)}{\tan^2(3x)} = \underbrace{\boxed{\frac{\sin(8x^2)}{8x^2}}}_{1} \; 8\cancel{x^2} \; \underbrace{\boxed{\frac{9x^2}{\tan^2(3x)}}}_{1} \; \frac{1}{9\cancel{x^2}} \rightarrow \frac{8}{9}$$

$$\sin(8x^2) = 8x^2 + o(x^2)$$

$$\tan(3x) = 3x + o(x)$$

$$\left[\tan(3x)\right]^2 = \left[3x + o(x)\right]^2 = 9x^2 + 6x\,o(x) + \left[o(x)\right]^2$$

$$= 9x^2 + o(x^2)$$

$$\frac{\sin(8x^2)}{\tan^2(3x)} = \frac{8x^2 + o(x^2)}{9x^2 + o(x^2)} = \frac{\cancel{x^2}\left(8 + \frac{o(x^2)}{x^2}\right)}{\cancel{x^2}\left(9 + \frac{o(x^2)}{x^2}\right)} \rightarrow \frac{8}{9}$$

$$\frac{\cos(3x)-1}{\arcsin(3x)} = \underset{-\frac{1}{2}}{\underset{\downarrow}{\boxed{\frac{\cos(3x)-1}{9x^2}}}} \cdot \underset{0}{\underset{\downarrow}{9x^2}} \cdot \underset{1}{\underset{\downarrow}{\boxed{\frac{3x}{\arcsin(3x)}}}} \cdot \frac{1}{3\cancel{x}} \to 0$$

$$\overset{f(x)\to 0}{\underset{1-A^3}{\boxed{\frac{1-\cos^2 x^3}{1-\cos^3 x^2}}}} = \frac{\overset{-0-0-}{(1+\cos x^3)(1-\cos x^3)}}{\underset{(1-A)}{(1-\cos x^2)}\underset{(1+A+A^2)}{(1+\cos x^2+\cos^2 x^2)}} =$$

$$= \underset{\frac{2}{3}}{\underset{\downarrow}{\boxed{\frac{1+\cos x^3}{1+\cos x^2+\cos^2 x^2}}}} \; \underset{\frac{1}{2}}{\underset{\downarrow}{\boxed{\frac{1-\cos x^3}{x^6}}}} \cdot \underset{0}{\underset{\downarrow}{\boxed{x^{\cancel{6}\,2}}}} \; \underset{2}{\underset{\downarrow}{\boxed{\frac{x^4}{1-\cos x^2}}}} \; \frac{1}{\cancel{x^4}}$$

$$\underset{0}{\underset{\downarrow}{\boxed{-f(x)}}} \le \underset{\to 0}{\boxed{f(x)\cdot\cos\frac{1}{x}}} \le \underset{0}{\underset{\downarrow}{\boxed{f(x)}}}$$

$$\underset{\downarrow\, +\infty}{\boxed{n^8 + (-1)^n n^5}} \geq \underset{\downarrow\, +\infty}{\boxed{n^8 - 27 n^5}}$$

$$\frac{\log\left(n^2+2\sqrt{n}\right)}{\log\left(n^3+3\sqrt[3]{n}\right)} = \frac{\log\left[n^2\left(1+\frac{2}{n\sqrt{n}}\right)\right]}{\log\left[n^3\left(1+\frac{3}{n^{3-\frac{1}{3}}}\right)\right]} =$$

$$\frac{2\log n + \log\left(1+\frac{2}{n\sqrt{n}}\right)}{3\log n + \log\left(1+\frac{3}{n^{3-1/3}}\right)} =$$

$$= \frac{\cancel{\log n}\left[2 + \overset{\to 0}{\boxed{\frac{\log(1+\dots)}{\log n}}}\right]}{\cancel{\log n}\left[3 + \underset{\to 0}{\boxed{\frac{\log(1+\dots)}{\log n}}}\right]} \longrightarrow \frac{2}{3}$$

$$\frac{\log\left(n^2+2\sqrt{n}\right)}{\log\left(n^3+3\sqrt[3]{n}\right)} \sim \frac{\log n^2}{\log n^3} = \frac{2\,\cancel{\log n}}{3\,\cancel{\log n}} \to \frac{2}{3}$$