MATEMATICA I

ORA 20

Titolo nota 12/10/2007

Esercizio 1 $\lim_{x \to +\infty} (3+\cos x)^x = +\infty$ perchè ?

$$3+\cos x \geq 2$$

$\cos x \geq -1$ aggiungo 3 a dx e sx

elevo alla x-esima (che è un esponente > 0)

$$\boxed{(3+\cos x)^x} \geq \boxed{2^x}$$

$\downarrow +\infty$ $\quad\quad$ $\downarrow +\infty$

confronto a 2.

Esercizio 2 $\lim_{x \to \boxed{+\infty}} (2+\cos x)^x$ come prima sarebbe

$$(2+\cos x)^x \geq \boxed{1^x} \to 1$$

Questo limite NON esiste !!!! Posto $f(x) = (2+\cos x)^x$

devo trovare 2 successioni $a_n \to \boxed{+\infty}$ $\quad b_n \to \boxed{+\infty}$

t.c. $f(a_n) \to l_1 \qquad f(b_n) \to l_2$

$a_n = 2\pi n \to +\infty$; $f(a_n) = (2+\cos(2\pi n))^{2\pi n}$

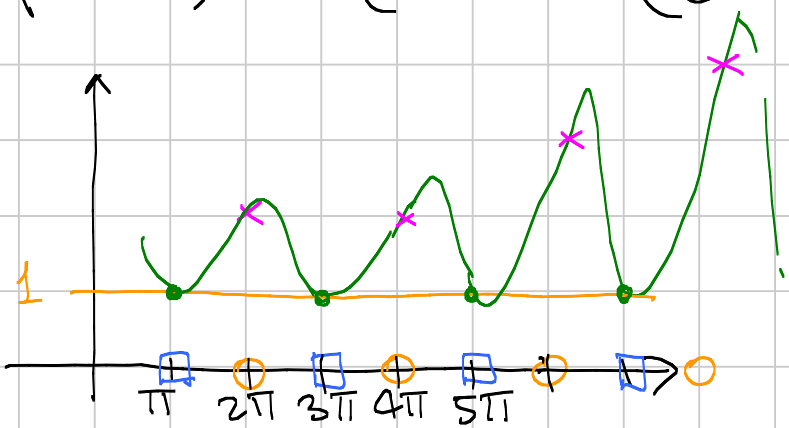
$= (2+1)^{2\pi n} = 3^{2\pi n} \to \boxed{+\infty}$

$b_n = \pi + 2\pi n = (2n+1)\pi$

$f(b_n) = (2+\cos((2n+1)\pi))^{(2n+1)\pi} = (2-1)^{(2n+1)\pi}$

$= 1^{\text{qualunque cosa}} \to \boxed{1}$

DIVERSI



□ = b_n

○ = a_n

Esercizio 3

$$\frac{\sin(n+\arctan n)}{n+\arctan n} = $$

Pongo $x = n+\arctan n$
Quando $n \to +\infty$, ho che $x \to +\infty$

$$= \lim_{x\to+\infty} \frac{\sin x}{x} = 0$$

Esempio 4

$$\lim_{x\to 0} \frac{\sin(x+\arctan x)}{x+\arctan x}$$

Pongo $y = x+\arctan x$
Quando $x \to 0$, ho che $y \to 0$

$$= \lim_{y\to 0} \frac{\sin y}{y} = 1.$$

Esercizio 5

$$\frac{n\sin(n^2) + n^2 \sin n}{(n+1)(n+\sqrt{n^3})} \sim \frac{n^2 \sin n}{n \cdot n^{\frac{3}{2}}} \sim \frac{\sin n}{n^{1/2}} \to 0$$

Rigorosamente: raccolgo n^2 al numeratore
e " n e $n^{3/2}$ nei 2 termini del den.

$$\frac{n^2\left(\boxed{\frac{\sin(n^2)}{n}}+\sin n\right)}{n\left(1+\boxed{\frac{1}{n}}\right)n^{3/2}\left(\boxed{\frac{1}{\sqrt{n}}}+1\right)}$$

FINALE COME SEMPRE

Esercizio 6 $\sqrt[n]{4^n+n^4}-\sqrt[n]{3^n+n^3}$ $\sim \sqrt[n]{4^n}-\sqrt[n]{3^n}=4-3=1$

Rigoroso $\sqrt[n]{4^n\left(1+\frac{n^4}{4^n}\right)}-\sqrt[n]{3^n\left(1+\frac{n^3}{3^n}\right)}$

$$4\,\underbrace{\boxed{\left(1+\frac{n^4}{4^n}\right)^{\frac{1}{n}}}}_{1^0=1}-3\,\underbrace{\boxed{\left(1+\frac{n^3}{3^n}\right)^{\frac{1}{n}}}}_{1^0=1}\longrightarrow 4-3=1.$$

Esercizio 7 $\lim_{x \to 0} \frac{\log(1+x^4)}{x}$ $\left[\frac{0}{0}\right]$

$$\frac{\log(1+x^4)}{x^4} \cdot \frac{x^4}{x} = \underbrace{\boxed{\frac{\log(1+x^4)}{x^4}}}_{\downarrow\, 1} \cdot \underbrace{\boxed{x^3}}_{\downarrow\, 0} \to 0$$

Esercizio 8 $\lim_{x \to 0} (\cos x)^{\frac{1}{\sin x^2}}$ $[1^\infty]$ $= e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

$$(\cos x)^{\frac{1}{\sin x^2}} = e^{\frac{1}{\sin x^2} \cdot \log(\cos x)}$$

Basta fare il limite dell'esponente

$$\frac{\log(\cos x)}{\sin x^2} = \underbrace{\boxed{\frac{\log[1+(\cos x-1)]}{\cos x - 1}}}_{\downarrow\, 1} \cdot \underbrace{\boxed{\frac{\cos x - 1}{x^2}}}_{\downarrow\, -\frac{1}{2}} \cdot \underbrace{\boxed{\frac{x^2}{\sin x^2}}}_{\downarrow\, 1} \to -\frac{1}{2}$$

$y = \cos x - 1$

$\lim_{y \to 0} \frac{\log(1+y)}{y}$

Quando $x \to 0$, ho che $y \to 0$

Esercizio 9 $\lim_{x \to -\infty} \sqrt{x^2+x+3} + x$ $\quad \infty - \infty$

Pongo $y = -x$ Quando $x \to -\infty$, ho che $y \to +\infty$

$$= \lim_{y \to +\infty} \sqrt{y^2 - y + 3} - y \qquad \infty - \infty$$

$$\sqrt{y^2-y+3} - y \; \frac{\sqrt{} + y}{\sqrt{} + y} = \frac{\cancel{y^2} - y + 3 - \cancel{y^2}}{\sqrt{y^2-y+3} + y} \sim \frac{-y}{\sqrt{y^2} + y} \to -\frac{1}{2}$$

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Esercizio 10

$$\left(\log(1+n^2) - \log n\right) \boxed{\arctan n!} = +\infty$$

$\infty - \infty$

$$\log\left(\frac{1+n^2}{n}\right) = \log\left(n + \frac{1}{n}\right) \to +\infty$$

$$\text{"}\arctan(+\infty)\text{"} = \frac{\pi}{2}$$

Esercizio 11

$$\lim_{x \to \frac{\pi}{2}} \underbrace{(2x-\pi)}_{\downarrow \atop 0} \underbrace{\tan x}_{\downarrow \atop \pm\infty} =$$

Pongo $y = x - \frac{\pi}{2}$ Quando $x \to \frac{\pi}{2}$ ho che $y \to 0$

$x = y + \frac{\pi}{2}$

$$= \lim_{y \to 0} \left(2y + \cancel{\pi} - \cancel{\pi}\right) \tan\left(y + \frac{\pi}{2}\right) = \lim_{y \to 0} 2y \tan\left(y + \frac{\pi}{2}\right)$$

$$\tan\left(y + \frac{\pi}{2}\right) \overset{\text{FORMULA}}{=} -\frac{1}{\tan y} \qquad = \lim_{y \to 0} 2y \left(-\frac{1}{\tan y}\right) =$$

$$= -2 \lim_{y \to 0} \frac{y}{\tan y} = -2.$$