RICEVIMENTO

Titolo nota 30/10/2008

1 $$n^{n!} - (n!)^n = \underbrace{\boxed{n^{n!}}}_{\downarrow\ +\infty}\left(1 - \underbrace{\boxed{\frac{(n!)^n}{n^{n!}}}}_{\downarrow ?\ 0}\right) \to +\infty$$

$$n! \le n^n \qquad \underbrace{\boxed{0}}_{\downarrow\ 0} \le \underbrace{\boxed{\frac{(n!)^n}{n^{n!}}}}_{\downarrow\ 0} \le \frac{(n^n)^n}{n^{n!}} = \frac{n^{n^2}}{n^{n!}} = \underbrace{\boxed{\frac{1}{n^{n!-n^2}}}}_{\downarrow\ 0}$$

$$n! - n^2 = \underbrace{\boxed{n!}}_{\downarrow\ +\infty}\left(1 - \underbrace{\boxed{\frac{n^2}{n!}}}_{\downarrow\ 0}\right) \to +\infty$$

2 $$\binom{3n}{n} - \binom{3n}{n+1} = \underbrace{\boxed{\binom{3n}{n}}}_{\downarrow\ +\infty \text{ con criterio rapporto}}\left[1 - \overbrace{\boxed{\frac{\binom{3n}{n+1}}{\binom{3n}{n}}}}^{2}\right] \to -\infty$$

Da capire:

$$\frac{\binom{3n}{n+1}}{\binom{3n}{n}} = \frac{\cancel{(3n)!}}{(n+1)\cancel{!}\,\cancel{(2n-1)}!} \cdot \frac{\cancel{n!}\,(2n)\cancel{!}}{\cancel{(3n)!}} = \frac{2n}{n+1} \to 2$$

3 $$\left(\cos\frac{1}{n} - \sqrt[n]{2}\right)^n \quad [= 0^\infty] \quad \to 0.$$

4 $$\sqrt[n]{n^{\log n}} = \left[n^{\log n}\right]^{\frac{1}{n}} = n^{\frac{1}{n}\log n} = e^{\frac{\log^2 n}{n}} \to e^0 = 1$$

5 $$\lim_{x \to 0} \frac{1}{x^4}\left\{1 - \left(\frac{\sin x}{x}\right)^{x^2}\right\} = (\star) \qquad \log(1+t) = t + o(t)$$

$$\left(\frac{\sin x}{x}\right)^{x^2} = e^{x^2 \log\left(\frac{\sin x}{x}\right)} = e^{x^2 \log\left(1 - \frac{x^2}{6} + o(x^2)\right)} = e^{x^2\left(-\frac{x^2}{6} + o(x^2)\right)}$$

$$= e^{-\frac{x^4}{6} + o(x^4)} \overset{e^t = 1+t+o(t)}{=} 1 - \frac{x^4}{6} + o(x^4)$$

$$(\star) = \frac{1}{x^4}\left\{\cancel{1} - \cancel{1} + \frac{x^4}{6} + o(x^4)\right\} = \frac{1}{6} + \frac{o(x^4)}{x^4} \to \frac{1}{6}$$

$\boxed{6}$ $\lim\limits_{x \to 0} \sqrt[x]{\dfrac{4^x + 9^x}{2}}$

$$4^x = e^{x \log 4} = 1 + x \log 4 + o(x)$$
$$9^x = \dots = 1 + x \log 9 + o(x)$$

$$\frac{4^x + 9^x}{2} = \frac{2 + x \log 4 + x \log 9 + o(x)}{2} = 1 + x\, \frac{\log 4 + \log 9}{2} + o(x)$$
$$= 1 + x \log 6 + o(x)$$

$$\sqrt[x]{\frac{4^x + 9^x}{2}} = \left(1 + x \log 6 + o(x)\right)^{\frac{1}{x}} = e^{\frac{1}{x} \log(1 + x \log 6 + o(x))} \overset{\log(1+t) = t + o(t)}{=}$$
$$= e^{\frac{1}{x}(x \log 6 + o(x))} = e^{\log 6 + \frac{o(x)}{x}} = e^{\log 6} = 6.$$

— o — o —

Usando solo limiti notevoli...

$$\sqrt[x]{\frac{4^x + 9^x}{2}} = e^{\frac{1}{x} \log\left(\frac{4^x + 9^x}{2}\right)}$$ basta fare il limite dell'esponente

$$\frac{1}{x} \log\left(1 + \frac{4^x + 9^x}{2} - 1\right) = \frac{1}{x}\, \frac{\log\left(1 + \frac{4^x + 9^x}{2} - 1\right)}{\frac{4^x + 9^x}{2} - 1}\, \frac{4^x + 9^x - 2}{2} =$$

$$= \boxed{\frac{\log\left(1 + \boxed{\frac{4^x + 9^x}{2} - 1}\right)}{\underset{t}{\boxed{\frac{4^x + 9^x}{2} - 1}}}} \cdot \left(\boxed{\frac{4^x - 1}{2x}} + \boxed{\frac{9^x - 1}{2x}}\right) \to \log 6$$

$t \to 0$ $\dfrac{\log(1+t)}{t} \to 1$

$\dfrac{1}{2} \log 4 + \dfrac{1}{2} \log 9 = \log 6$

$\boxed{7}$ $\lim\limits_{x \to 0} \dfrac{\log(1 + \sin^2 x) - x^2}{\sin^2(\tan^2 x)}$ $\qquad \sin^2(\tan^2 x) = x^4 + o(x^4)$

$$\log(1 + \sin^2 x) = \sin^2 x - \frac{1}{2} \sin^4 x + o(x^4)$$
$$\log(1+t) = t - \frac{t^2}{2} + o(t^2)$$
$$= \left(x - \frac{x^3}{6}\right)^2 - \frac{1}{2} x^4 + o(x^4) = x^2 - \frac{x^4}{3} - \frac{1}{2} x^4 + o(x^4)$$
$$= x^2 - \frac{5}{6} x^4 + o(x^4)$$

$$\frac{\text{Num}}{\text{Den}} = \frac{\cancel{x^2} - \frac{5}{6} x^4 + o(x^4) - \cancel{x^2}}{x^4 + o(x^4)} \to -\frac{5}{6}$$

$\boxed{8}$ $\lim_{x\to 0} \left\{ \tan\left(\frac{\arccos x}{2}\right)\right\}^{\frac{1}{x}}$ $\qquad \arccos 0 = \frac{\pi}{2} \qquad \tan\frac{\pi}{4} = 1$

$[= 1^{\pm\infty}]$

$$\arccos x = \frac{\pi}{2} - x + o(x) \qquad f(0) = \frac{\pi}{2},\ f'(0) = -1$$

$$f(x) = f(0) + \frac{f'(0)}{1}x + o(x) \qquad f\left(\frac{\pi}{4}+t\right) = f\left(\frac{\pi}{4}\right) + f'\left(\frac{\pi}{4}\right)t + o(t)$$

$$\frac{\arccos x}{2} = \frac{\pi}{4} - \frac{1}{2}x + o(x) \qquad \tan\left(\frac{\pi}{4}+t\right) = 1 + 2t + o(t)$$

$$\tan\left(\frac{\arccos x}{2}\right) = \tan\left(\frac{\pi}{4} \underbrace{-\frac{1}{2}x + o(x)}_{t}\right) = 1 - x + o(x)$$

$$\{\dots\}^{\frac{1}{x}} = (1 - x + o(x))^{\frac{1}{x}} \to \frac{1}{e}$$

$$\downarrow$$

$$e^{\frac{1}{x}\log(1-x+o(x))} = e^{\frac{1}{x}(-x+o(x))} \to e^{-1}$$

—o—o—

Con limiti notevoli $= e^{\frac{1}{x}\log\left(\tan\left(\frac{\arccos x}{2}\right)\right)}$ basta vedere l'esponente!

$$\frac{1}{x}\underbrace{\boxed{\frac{\log\left(1+\boxed{\tan\left(\frac{\arccos x}{2}\right)-1}\right)}{\boxed{\tan\left(\frac{\arccos x}{2}\right)-1}=t}}}_{\to 1}\left[\tan\left(\frac{\arccos x}{2}\right)-1\right]$$

$\Rightarrow$ basta fare il limite di $\frac{1}{x}\left[\tan\left(\frac{\arccos x}{2}\right)-1\right] =$

$$= \frac{1}{x}\frac{\left[\tan\left(\frac{\pi}{4}+\boxed{\frac{\arccos x}{2}-\frac{\pi}{4}}\right)-1\right]}{\boxed{\frac{\arccos x}{2}-\frac{\pi}{4}}\ t\to 0}\left(\frac{\arccos x}{2}-\frac{\pi}{4}\right)$$

$\lim_{t\to 0}\frac{\tan\left(\frac{\pi}{4}+t\right)-1}{t}$ si fa con formula di addizione della tangente

Resta $\frac{1}{2}\lim_{x\to 0}\frac{\arccos x - \frac{\pi}{2}}{x}$ = rapporto incrementale di $\arccos x$ in $x=0$

$$\to \frac{1}{2}(\arccos x)'\Big|_{x=0} = -\frac{1}{2}$$

Cosa utile: $\arccos x + \arcsin x = \frac{\pi}{2} \quad \forall x \in [-1,1]$

$\arcsin x = \alpha \qquad \cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha = x$

$$\frac{\pi}{2} - \alpha = \arccos x$$

— o —

$\boxed{9}$ $$\frac{\log x}{\log(\sin x^2)} = \frac{\log x}{\log\left(\frac{\sin x^2}{x^2} \cdot x^2\right)} = \frac{\log x}{\log\left(\frac{\sin x^2}{x^2}\right) + 2\log x} =$$

$$= \frac{\cancel{\log} x}{\cancel{\log} x\left[2 + \boxed{\frac{\log(\ )}{\log x}}\right]} \to \frac{1}{2}$$

1

$\downarrow 0$

$\boxed{10}$ $$\sqrt[4]{\frac{2m^2+3}{m^2+1}} = \sqrt[4]{\frac{2\cancel{m^2}\left(1+\frac{3}{2m^2}\right)}{\cancel{m^2}\left(1+\frac{1}{m^2}\right)}} = \sqrt[4]{2}\left(1+\frac{3}{2m^2}\right)^{\frac{1}{4}}\left(1+\frac{1}{m^2}\right)^{-\frac{1}{4}}$$

$(1+x)^\alpha \qquad (1+x)^\alpha$

$\boxed{11}$ $$\sqrt[3]{n^3+8n^2} - n = \sqrt[3]{n^3\left(1+\frac{8}{n}\right)} - n = n\left(1+\frac{8}{n}\right)^{\frac{1}{3}} - n$$

$$= n\left(1 + \frac{8}{3n} + o\left(\frac{1}{n}\right)\right) - n$$

$$= \cancel{n} + \frac{8}{3} + \frac{o\left(\frac{1}{n}\right)}{\frac{1}{n}} - \cancel{n} \to \frac{8}{3}$$

$\downarrow 0$