Formulario del corso di Statistica I
Ingegneria Gestionale
a.a. 2009/10

- $\overline{x} = \frac{x_1+...+x_n}{n}$, $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2$, $\widehat{Cov} = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)\left(y_i - \overline{y}\right)$, $r = \frac{\widehat{Cov}}{S_{x\grave{\imath}X}S_Y} = \frac{\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\overline{x})^2\sum_{i=1}^{n}(y_i-\overline{y})^2}}$. $\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2 = \left(\sum_{i=1}^{n} x_i^2\right) - n\overline{x}^2$, $\sum_{i=1}^{n}\left(x_i - \overline{x}\right)\left(y_i - \overline{y}\right) = \left(\sum_{i=1}^{n} x_i y_i\right) - n\overline{xy}$.

- $n! = n \cdot (n-1) \cdots 2 \cdot 1$, $0! = 1$. $\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n\cdot(n-1)\cdots(n-k+1)}{k!}$.

- $P\left(A|B\right) = \frac{P(A\cap B)}{P(B)}$, $P\left(A\cap B\right) = P\left(A|B\right)P\left(B\right)$. A, B indipendenti: $P\left(A\cap B\right) = P\left(A\right)P\left(B\right)$, $P\left(A|B\right) = P\left(A\right)$, $P\left(B|A\right) = P\left(B\right)$. $P\left(A\right) = \sum_k P\left(A|B_k\right)P\left(B_k\right)$. $P\left(B|A\right) = \frac{P(A|B)P(B)}{P(A)}$.

- X discreta, valori a_j, $P\left(X = a_j\right) = p_j$, allora $E\left[X\right] = \sum_j a_j p_j$, $E\left[g\left(X\right)\right] = \sum_j g\left(a_j\right)p_j$, $E\left[X^2\right] = \sum_j a_j^2 p_j$. $P\left(X\in A\right) = \sum_{i:a_i\in A} P\left(X = a_i\right) = \sum_{i:a_i\in A} p_i$. $X\in\mathbb{N}$, $P\left(X\le n\right) = \sum_{i=0}^{n} p_i$, $P\left(X\ge n\right) = \sum_{i=n}^{\infty} p_i$.

- X continua, densità $f\left(x\right)$, allora $E\left[X\right] = \int_{-\infty}^{\infty} x f\left(x\right)dx$, $E\left[g\left(X\right)\right] = \int_{-\infty}^{\infty} g\left(x\right)f\left(x\right)dx$, in particolare $E\left[X^2\right] = \int_{-\infty}^{\infty} x^2 f\left(x\right)dx$. $P\left(X\in A\right) = \int_A f\left(x\right)dx$.

- $Var\left[X\right] = \sigma_X^2 := E\left[\left(X-\mu_X\right)^2\right]$ dove $\mu_X = E\left[X\right]$. $Var\left[X\right] = E\left[X^2\right] - \mu_X^2$. $Cov\left(X,Y\right) = E\left[\left(X-\mu_X\right)\left(Y-\mu_Y\right)\right]$, $Cov\left(X,Y\right) = E\left[XY\right] - \mu_X\mu_Y$. $\rho\left(X,Y\right) = \frac{Cov(X,Y)}{\sigma_X\sigma_Y}$. $-1\le\rho\left(X,Y\right)\le 1$.

- $E\left[aX+bY+c\right] = aE\left[X\right] + bE\left[Y\right] + c$. $Var\left[X+Y\right] = Var\left[X\right] + Var\left[Y\right] + 2Cov\left(X,Y\right)$. $Var\left[aX\right] = a^2 Var\left[X\right]$. Standardizzazione di X: $\frac{X-\mu_X}{\sigma_X}$.

- X, Y indipendenti: $P\left(X\in A, Y\in B\right) = P\left(X\in A\right)P\left(Y\in B\right)$. Implica $E\left[XY\right] = E\left[X\right]E\left[Y\right]$, $Cov\left(X,Y\right) = 0$, $\rho\left(X,Y\right) = 0$, $Var\left[X+Y\right] = Var\left[X\right] + Var\left[Y\right]$.

- $F\left(x\right) = P\left(X\le x\right)$. $F\left(t\right) = \int_{-\infty}^{t} f\left(x\right)dx$. $F'\left(t\right) = f\left(t\right)$. $F\left(q_\alpha\right) = \alpha$.

- $\varphi\left(t\right) = E\left[e^{tX}\right]$, $\varphi'\left(0\right) = E\left[X\right]$, $\varphi''\left(0\right) = E\left[X^2\right]$; $\varphi_{aX}\left(t\right) = E\left[e^{taX}\right] = \varphi_X\left(at\right)$. X, Y indipendenti implica $\varphi_{X+Y}\left(t\right) = \varphi_X\left(t\right)\varphi_Y\left(t\right)$.

- $X\sim B\left(n,p\right)$: $P\left(X=k\right) = \binom{n}{k}p^k\left(1-p\right)^{n-k}$, $E\left[X\right] = np$, $Var\left[X\right] = np\left(1-p\right)$, $\sigma = \sqrt{np\left(1-p\right)}$, $\varphi\left(t\right) = \left(q+pe^t\right)^n$ dove $q = 1-p$. $X_1, ..., X_n \sim B\left(1,p\right)$ indipendenti implica $S = X_1 + ... + X_n \sim B\left(n,p\right)$.

- $X\sim\mathcal{P}\left(\lambda\right)$: $P\left(X=k\right) = e^{-\lambda}\frac{\lambda^k}{k!}$, $E\left[X\right] = \lambda$, $Var\left[X\right] = \lambda$, $\sigma = \sqrt{\lambda}$, $\varphi\left(t\right) = e^{\lambda\left(e^t-1\right)}$. Se $np_n = \lambda$ allora $\lim_{n\to\infty}\binom{n}{k}p_n^k\left(1-p_n\right)^{n-k} = e^{-\lambda}\frac{\lambda^k}{k!}$.

- $X \sim N\left(\mu, \sigma^2\right)$: $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$. $E[X] = \mu$, $Var[X] = \sigma^2$, $\varphi(t) = e^{\mu t} e^{\frac{t^2\sigma^2}{2}}$. X, Y gaussiane indipendenti, $a, b, c \in \mathbb{R}$ implica $aX + bY + c$ gaussiana. $X \sim N\left(\mu, \sigma^2\right)$ si può scrivere come $X = \sigma Z + \mu$, con $Z \sim N(0,1)$. $F_{\mu,\sigma^2}(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$. $\Phi(-x) = 1 - \Phi(x)$. $q_\alpha = -q_{1-\alpha}$. Soglie $\mu \pm \sigma q_\alpha$.

- $X \sim Exp(\lambda)$: $f(x) = \lambda e^{-\lambda x}$ per $x \geq 0$, zero per $x < 0$. $E[X] = \frac{1}{\lambda}$, $Var[X] = \frac{1}{\lambda^2}$, $\sigma = \frac{1}{\lambda}$, $\varphi(t) = \frac{\lambda}{\lambda - t}$ per $t < \lambda$. $F(x) = 1 - e^{-\lambda x}$ per $x \geq 0$, zero per $x < 0$.

- TLC: $P\left(\frac{X_1 + \ldots + X_n - n\mu}{\sqrt{n}\sigma} \in A\right) \approx P(Z \in A)$, con $Z \sim N(0,1)$.

- $\overline{X} = \frac{X_1 + \ldots + X_n}{n} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$. $E\left[S^2\right] = \sigma^2$. $\frac{S^2}{\sigma^2}(n-1) \sim \chi^2_{n-1}$.

- $\mu = \overline{X} \pm \frac{\sigma q_{1-\frac{\alpha}{2}}}{\sqrt{n}}$; $\mu = \overline{X} \pm \frac{S \cdot t^{(n-1)}_{1-\frac{\alpha}{2}}}{\sqrt{n}}$.

- $\left|\frac{\overline{x} - \mu_0}{\sigma}\sqrt{n}\right| > q_{1-\frac{\alpha}{2}}$. $\left|\frac{\overline{x} - \mu_0}{S}\sqrt{n}\right| > t^{(n-1)}_{1-\frac{\alpha}{2}}$. $P\left(|Z| > \left|\frac{\overline{x} - \mu_0}{S}\sqrt{n}\right|\right)$, $P_\mu\left(\overline{X} \in \left[\mu_0 - \frac{\sigma q_{1-\frac{\alpha}{2}}}{\sqrt{n}}, \mu_0 + \frac{\sigma q_{1-\frac{\alpha}{2}}}{\sqrt{n}}\right]\right)$. $\frac{S^2}{\sigma^2}(n-1) > \chi^2_{\alpha, n-1}$. $T = n\sum_{i=1}^{k} \frac{(\widehat{p}_i - p_i)^2}{p_i} = \sum_{i=1}^{k} \frac{\left(\widehat{X}_i - np_i\right)^2}{np_i} > \chi^2_{\alpha, k-1}$.

- $y = A + Bx$, $B = \frac{\widehat{Cov}}{S_X^2} = r\frac{S_Y}{S_X}$, $\overline{y} = A + B\overline{x}$.