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Lezioni di Analisi Matematica 2 e Complementi

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Ricevimento su appuntamento da concordare per email

$$I = \iiint_D \frac{z}{(x^2+y^2)^\alpha}\,dx\,dy\,dz \qquad D = \{ 0 \le z \le 1 ,\ x^2+y^2 \le 1/z^2 \},\ \alpha \in \mathbb{R}$$

(se $z=0$ ogni (x,y) è o.k.)

· GIÀ VISTO CHE $f(x,y,z) = \dfrac{z}{(x^2+y^2)^\alpha}$ è continua su $\mathbb{R}^3 \setminus E$

$E = \{ x^2+y^2 = 0 \} = \{ (0,0,z) : z \in \mathbb{R} \}$ = "asse z"

$m(E) = 0 \Rightarrow f$ (continua eccetto che su un trascurabile) è misurabile

($\Rightarrow$ POSSO USARE TONELLI – $f \ge 0$ su D)

DUNQUE I esiste in $[0,+\infty]$ – NOTA CHE

$$I = \iiint_{\mathbb{R}^N} \tilde{f}(x,y,z)\,dx\,dy\,dz \quad \text{dove} \quad \tilde{f} = \begin{cases} f & \text{in } D \\ 0 & \text{fuori da } D \end{cases}$$

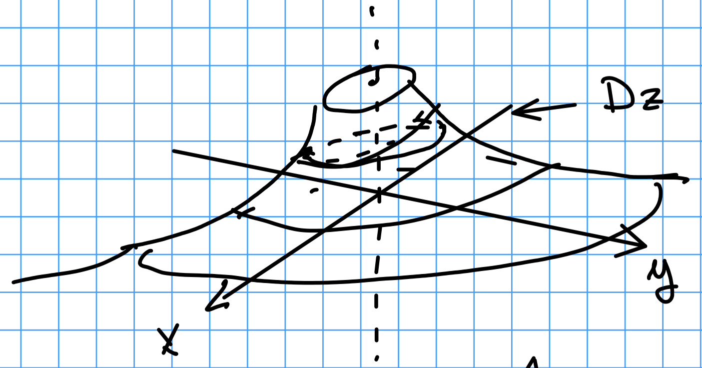
$\tilde{f}$ è mis, $\tilde{f} \ge 0 \Rightarrow I$ esiste in $[0,+\infty]$. POSSO USARE TONELLI rispetto alla decomposizione $\mathbb{R}^3 = \mathbb{R}_z \times \mathbb{R}^2_{xy} \Rightarrow$

$$I = \int_{\mathbb{R}} \left(\int_{\mathbb{R}^2} \tilde{f}(x,y,z)\,dx\,dy \right) dz$$

NB. $D_z = \{ (x,y) : (x,y,z) \in D \} = \begin{cases} \emptyset & \text{se } z \notin [0,1] \\ B(0, 1/z^2) & \text{se } z \in [0,1] \end{cases}$

(se $z=0$ $B(0,1/z^2) = \mathbb{R}^2$

NB. $D \rightarrow$



ALLORA

$$I = \int_0^1 \left(\iint_{D_z} f(x,y,z)\right) dz = \int_0^1 z \iint_{B(0,1/z^2)} \frac{dx\,dy}{(x^2+y^2)^\alpha}$$

$$\iint_{B(0,1/z^2)} \frac{dx\,dy}{(x^2+y^2)^\alpha} =$$ si può fare più semplicemente (coord. polari)

$$\int_{B_{xz}} \left(\int \frac{1}{(x^2+y^2)^\alpha}\, dy\right) dx =$$

$B_{xz} = \{ y : y^2 + x^2 \le 1/z^2 \} = \{ y : y^2 \le 1/z^2 - x^2 \} = [-\sqrt{1/z^2 - x^2}, \sqrt{1/z^2 - x^2}]$

$(= \emptyset$ se $x^2 > 1/z^2)$

$$\int_{-\frac{1}{z}}^{\frac{1}{z}} \underbrace{\left(\int_{-\sqrt{\frac{1}{z^2}-x^2}}^{\sqrt{1/z^2-x^2}} \frac{dy}{(x^2+y^2)^\alpha} \right)}_{(*)} dx$$

$$(*) = 2\int_0^{\sqrt{1/z^2-x^2}} \frac{dy}{(x^2+y^2)^\alpha}$$ ← INT. SECONDO R. — LO CALCOLO COME AL SOLITO

conviene fare un cambio di var. $y = t|x| \Rightarrow$

t varia tra $\bar t$ dove $\bar t^2 x^2 = \frac{1}{z^2} - x^2 \iff x^2(\bar t^2 + 1) = \frac{1}{z^2}$

$\bar t^2 = \frac{1}{x^2 z^2} - 1$, $dy = |x|\,dt$ dunque viene

$$2\int_0^{\sqrt{\frac{1}{x^2z^2}-1}} |x| \left(x^2 + t^2 x^2\right)^\alpha dt = 2|x|^{1+2\alpha} \int_0^{\sqrt{\frac{1}{x^2z^2}-1}} (1+t^2)^\alpha dt$$

COMPLICATO — RINUNCIO A CONTINUARE —

USIAMO LE COORD. POLARI PER CALCOLARE l'int. in xy

COOR. POL. Se $\phi(\rho,\theta) = \begin{pmatrix} \rho\cos\theta \\ \rho\sin\theta \end{pmatrix} \Rightarrow J_\phi = \begin{bmatrix} \cos\theta & -\rho\sin\theta \\ \sin\theta & \rho\cos\theta \end{bmatrix}$

$$\Rightarrow \det J_\phi = \rho \det \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \rho$$

DUNQUE $\int_A f(\rho\cos\theta, \rho\sin\theta)\,\rho\, d\rho\, d\theta = \int_{\phi(A)} f(x,y)\, dx\, dy$ (*)

ANDREBBE PRECISATO CHE $A \subset [0,+\infty[\times [0,2\pi]$ o $[0,+\infty] \times [a,b]$

CON $b-a = 2\pi$. A RIGORE Φ NON È BIGETTIVA, MA

I PUNTI IN CUI NON È INIETTIVA SONO TRASCURABILI

($\{\rho=0 \;\; 0 \le \theta \le 2\pi\} \cup \{\rho \ge 0, \;\; \theta = 0, 2\pi\}$). A CAUSA DI CIÒ

LA FORMULA (*) FUNZIONA!

Se voglio calcolare

$$\iint_{B(0,1/2^2)} \frac{dx\,dy}{(x^2+y^2)^\alpha} = \iint_{\substack{0 \le \rho \le 1/2^2 \\ 0 \le \theta \le 2\pi}} \frac{\rho\, d\rho\, d\theta}{\rho^{2\alpha}} \overset{\text{FUBINI}}{=} 2\pi \int_0^{1/2^2} \rho^{1-2\alpha}\, d\rho =$$

$$2\pi \frac{1}{2-2\alpha} \rho^{2-2\alpha} \Big|_0^{1/2^2} \quad \text{SE } 2-2\alpha > 0 \;\; \boxed{\alpha < 1}$$

(INT. IMPROPRIO DI R. CON $\delta \ge 0$)

$$= +\infty \quad \text{SE } 2-2\alpha \le 0 \;\; \boxed{\alpha \ge 1}$$

(nel caso $2-2\alpha=0$ c'è un log.)

$$\iint_{B(0,1/2^2)} \frac{dx\,dy}{(x^2+y^2)^\alpha} = \begin{cases} +\infty & \text{se } \alpha \ge 1 \\ \dfrac{\pi}{1-\alpha} \dfrac{1}{z^{4(1-\alpha)}} & \end{cases}$$

FACCIAMO L'INT. IN z!

$$\int_0^1 z \bigcirc \, dz = \begin{cases} +\infty & \text{se } \alpha \ge 1 \\ \displaystyle\int_0^1 z^{1-4+4\alpha}\, dz & \alpha < 1 \end{cases}$$

$\left(\int_0^1 +\infty \, dz = +\infty \; ! \right)$

$\boxed{\text{CASO } \alpha < 1}$

$$\int_0^1 \frac{dz}{z^{4(\alpha-1)}} = \int_0^1 z^{4\alpha-3}\, dz = \begin{cases} \dfrac{1}{4\alpha-2} \left[z^{4\alpha-2} \right]_0^1 & \text{se } 4\alpha-2 > 0 \\ +\infty & \text{se } 4\alpha-2 \le 0 \end{cases}$$

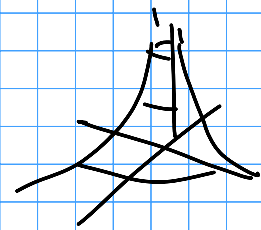
$$= \begin{cases} +\infty & \text{se } \alpha \le 1/2 \\ \dfrac{1}{4\alpha-2} & \text{se } \alpha > 1/2 \end{cases}$$

ALLA FINE

$$\iiint_D \delta = \begin{cases} +\infty & \text{SE } \alpha \le 1/2 \text{ oppure } \alpha \ge 1 \\ \dfrac{\pi}{(4\alpha-2)(1-\alpha)} & \text{se } \dfrac{1}{2} < \alpha < 1 \end{cases}$$

ESEMPIO

$$\int_{B(0,1)} \frac{dx\,dy}{(x^2+y^2)^\alpha}$$



COORD. POLARI

$$\int_0^{2\pi}\left(\int_0^1 \frac{\rho\, d\rho}{\rho^{2\alpha}}\right)d\theta = 2\pi \int_0^1 \rho^{1-2\alpha}\, d\rho = \begin{cases} +\infty & \text{se } 2-2\alpha \le 0 \\ \frac{2\pi}{2-2\alpha}\left[\rho^{2-2\alpha}\right]_0^1 & \text{SE } 2-2\alpha > 0 \end{cases}$$

$$= \begin{cases} +\infty & \text{se } \alpha \geq 1 \\ \frac{\pi}{1-\alpha} & \text{se } \alpha < 1 \end{cases}$$

N.B. L'INTEGRANDO È $\frac{1}{\|(x,y)\|^{2\alpha}}$

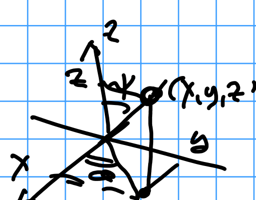
DUNQUE
$$\int_{B(0,1)} \frac{dx\,dy}{\|(x,y)\|^\beta} \begin{cases} < +\infty & \text{se } \beta < 2 \\ +\infty & \text{se } \beta \geq 2 \end{cases}$$

ES.
$$\iiint_{B(0,1)} \frac{dx\,dy\,dz}{(x^2+y^2+z^2)^{\alpha/2}} = \iiint_{B(0,1)} \frac{dx\,dy\,dz}{\|(x,y,z)\|^\alpha}$$

COORDINATE SFERICHE:

$$\phi(\rho,\theta,\psi) = (\rho\cos\theta\sin\psi,\ \rho\sin\theta\sin\psi,\ \rho\cos\psi)$$

ψ è l'angolo tra l'asse z e il vett. $\overrightarrow{(x,y,z)}$



$$J_\phi = \begin{bmatrix} \cos\theta\sin\psi, & -\rho\sin\theta\sin\psi, & \rho\cos\theta\cos\psi \\ \sin\theta\sin\psi, & \rho\cos\theta\sin\psi, & \rho\sin\theta\cos\psi \\ \cos\psi, & 0, & -\rho\sin\psi \end{bmatrix}$$

$$\det J_\phi = \rho^2\left(\cos\psi \det\begin{bmatrix} -\sin\theta\sin\psi & \cos\theta\cos\psi \\ \cos\theta\sin\psi & \sin\theta\cos\psi \end{bmatrix} - \sin\psi \det\begin{bmatrix} \cos\theta\sin\psi & -\sin\theta\sin\psi \\ \sin\theta\sin\psi & \cos\theta\sin\psi \end{bmatrix}\right) =$$

$$\rho^2\Big(\cos^2\psi\,\sin\psi\;\underbrace{\det\begin{bmatrix}-\sin\theta & \cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}}_{-1} - \sin^3\psi\;\det\underbrace{\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}}_{1}\Big)$$

$$-\rho^2\sin\psi\left(\cos^2\psi+\sin^2\psi\right) = -\rho^2\sin\psi$$

Sto considerando $\rho \geq 0$ $\quad 0 \leq \theta \leq 2\pi$ $\quad 0 \leq \psi \leq \pi$ $\quad \Rightarrow \sin\psi \geq 0$

$|\det J_\phi| = \rho^2 \sin\psi$ DUNQUE (MODULO QUALCHE PRECISAZIONE)

$$\iiint_{\phi(A)} f(x,y,z)\,dx\,dy\,dz = \iiint_A f(\phi(\rho,\theta,\psi))\,\rho^2\sin\psi\,d\rho\,d\theta\,d\psi$$

TORNANDO A $\displaystyle\iiint_{B(0,1)} \frac{dx\,dy\,dz}{\|(x,y,z)\|^\alpha} = \int_0^\pi\left(\int_0^{2\pi}\left(\int_0^1 \frac{\rho^2\sin\psi\,d\rho}{\rho^\alpha}\right)d\theta\right)d\psi =$

$$2\pi\underbrace{\int_0^\pi \sin\psi\,d\psi}_{2}\int_0^1 \rho^{2-\alpha}\,d\rho = 4\pi\int_0^1 \rho^{2-\alpha}\,d\rho = \begin{cases} +\infty & \text{se } 3-\alpha\leq 0\\ \dfrac{4\pi}{3-\alpha}\,\rho^{3-\alpha}\Big|_0^1 & \text{se } 3-\alpha>0\end{cases}$$

$$= \begin{cases} +\infty & \text{se } \alpha \geq 3\\ \dfrac{4\pi}{3-\alpha} & \text{se } \alpha < 3\end{cases}$$

SI POTREBBE DIMOSTRARE CHE IN $\mathbb{R}^N$

$$\int_{B(0,1)} \frac{1}{\|x\|^\alpha}\,dx \begin{cases} +\infty & \text{se } \alpha \geq N\\ <+\infty & \text{se } \alpha < N \to \dfrac{\omega_N}{N-\alpha}\end{cases}$$

dove ω_N = "misura $N-1$ dimensionale" della sfera unitaria

$$I = \iiint_D xyz\,dx\,dy\,dz \qquad D = \{x^2+y^2+z^2\leq 1,\ x\geq 0,\ y\geq 0,\ z\geq 0\}$$

MODO "BRUTALE" USO TONELLI (due volte)

$$I = \int\Big(z\iint_{D_z} xy\,dx\,dy\Big)\,dz$$

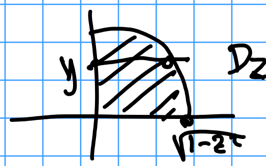
$D_z = \{(x,y) : (x,y,z)\in D\} =$

$\{x^2+y^2 \leq 1-z^2,\ x\geq 0,\ y\geq 0\}$ se $z\geq 0$

ϕ se $z<0$

$D_z = \phi$ se $z<0$ oppure $z>1$

$$I = \int_0^1 z \left(\iint_{B(0,\sqrt{1-z^2})} xy \, dx\,dy \right) dz =$$



$$= \int_0^1 z \left(\int_0^{\sqrt{1-z^2}} y \left(\int_0^{\sqrt{1-z^2-y^2}} x \, dx \right) dy \right) dz =$$

$$\int_0^1 z \left(\int_0^{\sqrt{1-z^2}} y \left[\frac{x^2}{2} \right]_0^{\sqrt{1-y^2-z^2}} dy \right) dz =$$

$$\frac{1}{2} \int_0^1 z \left(\int_0^{\sqrt{1-z^2}} y \left(1-y^2-z^2 \right) dy \right) dz =$$

$$\frac{1}{2} \int_0^1 z \left(\int_0^{\sqrt{1-z^2}} \left(y(1-z^2) - y^3 \right) dy \right) dz =$$

$$\frac{1}{2} \int_0^1 z \left[\frac{y^2}{2}(1-z^2) - \frac{y^4}{4} \right]_0^{\sqrt{1-z^2}} dz =$$

$$\frac{1}{8} \int_0^1 z \left(2(1-z^2)(1-z^2) - (1-z^2)^2 \right) dz =$$

$z^2 = s \qquad 2z\,dz = ds$

$$\frac{1}{8} \int_0^1 z \left(1-z^2\right)^2 dz =$$

$$\frac{1}{16} \int_0^1 (1-s)^2 \, ds = \frac{1}{16} \left[-\frac{(1-s)^3}{3} \right]_0^1 = \frac{1}{16} \cdot \frac{1}{3}$$

II° MODO (COORD SFERICHE)

$$\iiint_D xyz \, dx\,dy\,dz = \int_0^{\pi/2} \left(\int_0^{\pi/2} \left(\int_0^1 \cos\theta \sin^2\psi \sin\theta \cos\psi \; \rho^3 \; \rho^2 \sin\psi \, d\rho \right) d\theta \right) d\psi =$$

$$\underbrace{\int_0^{\pi/2} \sin^3\psi \cos\psi \, d\psi}_{1} \quad \underbrace{\int_0^{\pi/2} \cos\theta \sin\theta}_{2} \quad \underbrace{\int_0^1 \rho^5 \, d\rho}_{3}$$

1 $\quad s = \sin\psi \Rightarrow ds = \cos\psi \, d\psi$

$$\Rightarrow \int_0^1 s^3 \, ds = \left[\frac{s^4}{4} \right]_0^1 = \frac{1}{4}$$

2 $$= \frac{1}{2} \int_0^{\pi/2} \sin(2\theta) \, d\theta = \frac{1}{2} \left[-\frac{\cos(2\theta)}{2} \right]_0^{\pi/2} = \frac{1}{2} \cdot \frac{1+1}{2} = \frac{1}{2}$$

$$③ = \left[\frac{\rho^6}{6} \right]_0^1 = \frac{1}{6} \quad \Rightarrow \quad \text{INTEGRALE} = \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{16} \cdot \frac{1}{3}$$

TORNA

ESEMPIO (CALCOLO NOTEVOLE)

$$I = \int_{-\infty}^{+\infty} e^{-x^2}\, dx = ??$$

(è noto che non si riesce a esprimere la primitiva di e^{-x^2} in termini elementari)

TRUCCO

$$I^2 = \int_{-\infty}^{+\infty} e^{-x^2}\, dx \int_{-\infty}^{+\infty} e^{-y^2}\, dy \overset{\text{FUBINI (A RITROSO)}}{=} \iint_{\mathbb{R}^2} e^{-x^2} e^{-y^2}\, dx\, dy = \iint_{\mathbb{R}^2} e^{-(x^2+y^2)}\, dx\, dy$$

($f \geq 0$ f continua posso usare il cambio di var. in coord. polari — eventualmente posso trovare $+\infty$)

$$= \iint_{\{\rho \geq 0,\, 0 \leq \theta \leq 2\pi\}} e^{-\rho^2} \rho\, d\rho\, d\theta =$$

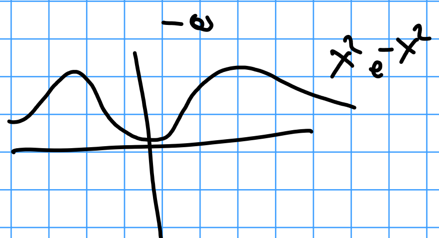
$$\left(\int_0^{2\pi} d\theta \right) \left(\int_0^{+\infty} \rho e^{-\rho^2}\, d\rho \right) = \qquad s = \rho^2 \qquad ds = 2\rho\, d\rho$$

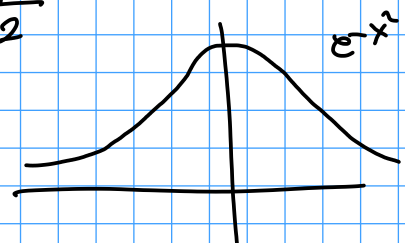
$$2\pi \, \frac{1}{2} \int_0^{+\infty} e^{-s}\, ds = \pi \left[-e^{-s} \right]_0^{+\infty} = \pi$$

$$\Rightarrow \int_{-\infty}^{+\infty} e^{-x^2}\, dx = \sqrt{\pi} \qquad \left(\int_0^{+\infty} e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2} \right)$$

$$\int_{-\infty}^{+\infty} x^2 e^{-x^2}\, dx = \int_{-\infty}^{+\infty} x \underbrace{\left(x e^{-x^2} \right)}_{-\frac{1}{2} \frac{d}{dx} e^{-x^2}} dx \overset{\text{per parti}}{=} \underbrace{\left[x \left(-\frac{1}{2} e^{-x^2} \right) \right]_{-\infty}^{+\infty}}_{\text{VINCE } e^{-x^2} \rightarrow \text{ VIENE } 0}$$

$$+ \frac{1}{2} \int_{-\infty}^{+\infty} e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}$$





$$\iint_{B(0,1)} \underbrace{\frac{xy}{x^2+y^2}}_{f(x,y)}\, dx\,dy$$ ← DIRE SE ESISTE, e in caso affermativo trovare quanto fa

ATTENZIONE

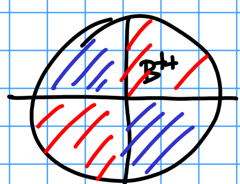
- f non è limitata $\Rightarrow$ NON POSSO DIRE IN MODO FACILE che f è integrabile.
- Sicuramente f è misurabile $\Leftarrow$ f è continua eccetto che in $(x,y)=(0,0)$

PERÒ f non è ≥ 0 (e neanche ≤ 0)

- Se sapessi che f è integrabile, POTREI DIRE SUBITO -USANDO LA SIMMETRIA- che $\iint_B f = 0$

PER VEDERE SE f è integrabile devo vedere se

$$\iint_B |f| < +\infty \iff \iint_{B^{++}} \frac{xy}{x^2+y^2}\, dx\,dy < +\infty$$



$B^{++} = \{x^2+y^2 \leq 1,\ x \geq 0, y \geq 0\}$

↑

VEDIAMO QUESTO

SO CHE ESISTE PERCHÈ $f \geq 0$ su B^{++}

Dato che $f \geq 0$ posso passare in coordinate polari

$$\iint_{B^{++}} \frac{xy}{x^2+y^2}\, dx\,dy = \int_0^{\pi/2}\int_0^1 \frac{\rho^2 \cos\theta \sin\theta}{\rho^2}\, \rho\, d\rho$$

$$= \int_0^{\pi/2} \cos\theta \sin\theta\, d\theta \int_0^1 \rho\, d\rho < +\infty \Rightarrow$$ INTEGRABILE e INTEGRALE $=0$

SE INVECE AVESSI

$$\iint_B \frac{xy}{(x^2+y^2)^2}\, dx\,dy \qquad B = \{x^2+y^2 \leq 1\}$$

↑

GUARDO SE È INTEGRABILE CALCOLANDO

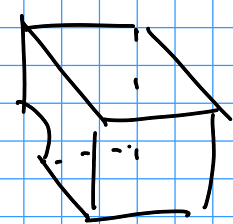
$$\iint_{B+} \frac{xy}{(x^2+y^2)^2}\,dx\,dy \underset{\text{polari}}{=} \int_0^{\pi/2}\int_0^1 \frac{\rho\cos\theta\,\rho\sin\theta\,\rho\,d\rho\,d\theta}{\rho^4} =$$

$$\underbrace{\int_0^{\pi/2}\cos\theta\sin\theta\,d\theta}_{>0\ (\text{finito})}\ \underbrace{\int_0^1 \frac{d\rho}{\rho}}_{=+\infty} = +\infty$$

f NON E' INTEGRABILE

ESERCIZIO

$$\iiint_D \underbrace{\frac{xy}{\sqrt{z}}}_{\geq 0 \ (\text{misurabile: continua tranne piano } xy)}\,dx\,dy\,dz \qquad D=\{x^2+y^2+z^2\geq 1,\ 0\leq x\leq 2,\ 0\leq y\leq 2,\ 0\leq z\leq 2\}$$



CONVIENE NOTARE CHE $D = Q\setminus B$

dove $Q=[0,2]\times[0,2]\times[0,2]$

$B=\{x^2+y^2+z^2,\ x\geq 0, y\geq 0, z\geq 0\}$ $\quad (B\subset Q)$

$$\Rightarrow \iiint_D \frac{xy}{\sqrt{z}} = \iiint_Q \frac{xy}{\sqrt{z}}\,dx\,dy\,dz - \iiint_B \frac{xy}{\sqrt{z}}\,dx\,dy\,dz$$

A PATTO CHE $\iiint_B < +\infty$

$$\iiint_Q \frac{xy}{\sqrt{z}}\,dx\,dy\,dz = \int_0^2 x\,dx\int_0^2 y\,dy\int_0^2\frac{dz}{\sqrt{z}} = \underbrace{\left[\frac{x^2}{2}\right]_0^2}_{2}\underbrace{\left[\frac{y^2}{2}\right]_0^2}_{2}\underbrace{\left[2\sqrt{z}\right]_0^2}_{2\sqrt{2}} = 8\sqrt{2}$$

$$\iiint_B \frac{xy}{\sqrt{z}} = (\text{polari})$$

$$\int_0^{\pi/2}\left(\int_0^{\pi/2}\left(\int_0^1 \frac{\rho^2\cos\theta\sin\theta\sin^2\psi\,\rho^2\sin\psi\,d\rho}{\sqrt{\rho\cos\psi}}\right)d\theta\right)d\psi =$$

$$\underset{1}{\int_0^{\pi/2}\sin\theta\cos\theta\,d\theta}\quad \underset{2}{\int_0^{\pi/2}\frac{\sin^3\psi}{\sqrt{\cos\psi}}\,d\psi}\quad \underset{3}{\int_0^1 \rho^{7/2}\,d\rho}$$

$$① = \frac{1}{2}\left[-\frac{\cos(2\theta)}{2}\right]_0^{\pi/2} = \frac{1}{2}$$

$$② = \int_0^{\pi/2} \frac{1-\cos^2\psi}{\sqrt{\cos\psi}} \sin\psi \qquad s = \cos\psi \quad ds = -\sin\psi\, d\psi$$

$$= \int_1^0 \frac{1-s^2}{\sqrt{s}}(-ds) = \int_0^1 \frac{1-s^2}{\sqrt{s}}\, ds = \left[2\sqrt{s} - \frac{2}{5}s^{5/2}\right]_0^1 =$$

$$= 2 - \frac{2}{5} = \frac{8}{5}$$

$$③ \int_0^1 \rho^{7/2}\, d\rho = \left[\frac{2}{9}\rho^{9/2}\right]_0^1 = \frac{2}{9}$$

$$\Rightarrow \iiint_B z = \frac{1}{\cancel{2}}\,\frac{8}{5}\,\frac{\cancel{2}}{9} = \frac{8}{45}$$

$$\text{INT.} = 8\sqrt{2} - \frac{8}{45}$$