$$f(x,y,z) = xy + z$$

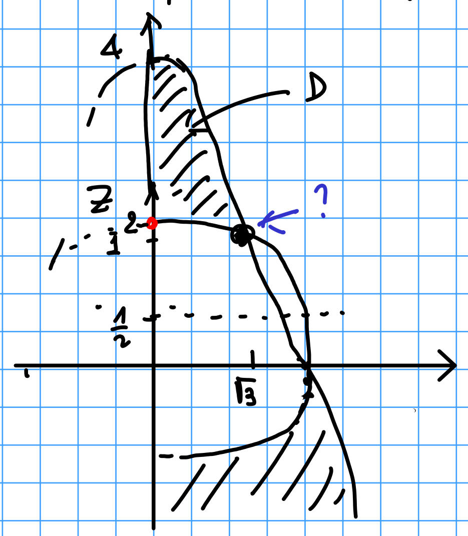
$$D = \{\ x^2+y^2+z^2 \geq 4,\quad 4-x^2-y^2 \geq z \geq 1/2\ \}$$

$G_1 = 4 - x^2 - y^2 - z^2$

$G_2 = z - 4 + x^2 + y^2$

$G_3 = 1/2 - z$

$D = \{ G_1 \leq 0,\ G_2 \leq 0,\ G_3 \leq 0 \}$



$\rho^2 + z^2 = 4$

$4 - \rho^2 = z$

$z^2 = 4 - \rho^2 = z$

caso $\quad G_1 = 0 \quad G_2 < 0 \quad G_3 < 0$ $\qquad$ 1 moltiplicatore

$\nabla f = \lambda \nabla G_1$

$$\nabla f = \begin{pmatrix} y \\ x \\ 1 \end{pmatrix} \qquad \nabla G_1 = -\begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$$

$$\begin{cases} y = -2\lambda x \\ x = -2\lambda y \\ 1 = -2\lambda z \\ \underbrace{x^2+y^2+z^2 = 4,}\ \underbrace{z < 4-x^2-y^2,}_{1 < 4-3=1 \text{ NO}}\ z > \frac{1}{2} \end{cases}$$

$\leftarrow \lambda \neq 0$

$y = -2\lambda x = -2\lambda(-2\lambda)y = 4\lambda^2 y$

$y = 0$ / $4\lambda^2 = 1$

$x = 0$

$z = \pm 2$

$$\boxed{\begin{matrix}(0,0,2) \\ \lambda = -1/4\end{matrix}}$$

$\lambda = \pm \frac{1}{2}$ $\qquad$ OK

$\lambda = \pm 1/2$

$z = \pm 1$

$x = y = \sqrt{3/2}$

$\underbrace{\left(\sqrt{\tfrac{3}{2}}, \sqrt{\tfrac{3}{2}}, 1\right)}$ $\quad$ NON VA

$y = \pm x$ $\quad \boxed{y = x}$

$\boxed{z = 1}$ $\quad \boxed{\lambda = -\frac{1}{2}}$

OPPURE $\quad$ moltiplico la I^a per x e la II^o per y $\Rightarrow$

$xy = -2\lambda x^2 \qquad xy = -2\lambda y^2 \Rightarrow \lambda x^2 = \lambda y^2 \qquad \lambda \neq 0$ per III^o

$\Rightarrow x^2 = y^2 \Rightarrow x = \pm y$

$x = y$ trovo $\quad \begin{cases} x = -2\lambda x \\ y = x \end{cases} \leftarrow \lambda = -\frac{1}{2},\ x = 0$

. . . .

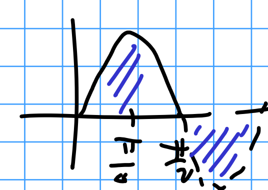
$f(x,y)=\dfrac{xy}{(x^2+y^2)^2}$ ← MISURABILE (CONTINUA ECCETTO CHE IN $(0,0)$)

$|f|$ ammette integrale ($|f|\geq 0$, $|f|$ mis.)

$D=\{x^2+y^2\leq 1\ ,\ y\geq 0\}$

$$\iint \left|\frac{xy}{(x^2+y^2)^2}\right| dx\,dy \overset{\text{polari}}{=} \int_0^{\pi} d\theta \int_0^1 \frac{\rho|\cos\theta|\rho|\sin\theta|}{\rho^4}\,\rho\, d\rho =$$

$$\int_0^{\pi}|\cos\theta \sin\theta|\,d\theta \int_0^1 \frac{d\rho}{\rho} \to +\infty$$



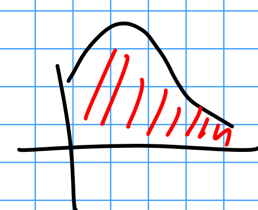
$$\int_0^{\pi}|\cos\theta\sin\theta|\,d\theta = \frac{1}{2}\int_0^{\pi}\sin 2\theta\, d\theta =$$

$$\frac{2}{2}\int_0^{\pi/2}\sin(2\theta)\,d\theta = \left[-\frac{\cos 2\theta}{2}\right]_0^{\pi/2} = 1$$

$$\int_0^{\pi/2}\cos\theta\sin\theta + \int_{\pi/2}^{\pi}-\cos\theta\sin\theta = 2\int_0^{\pi/2}\cos\theta\sin\theta = \int_0^{\pi/2}\sin(2\theta)\,d\theta$$

$$\boxed{\iint_D |f| = +\infty}$$

f NON È INTEGRABILE



DEF. Sia f misurabile

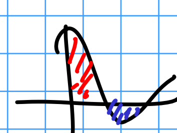
(a) se $f\geq 0$ f ammette integrale $=$ mis$\{(x,y):\ 0\leq y\leq f(x)\}$ |

$\int f\,dx$ ($\in[0,+\infty]$)

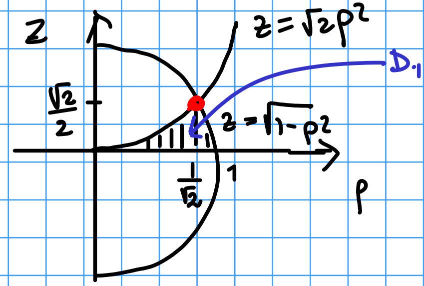
(b) se f NON È ≥ 0 dico che f è integrabile $\Longleftrightarrow$

$\int f^+ < +\infty$ e $\int f^- < +\infty$ e pongo

$\int f = \int f^+ - \int f^-$



$f = \frac{zx}{x^2+y^2}$ $\qquad D = \{ x^2+y^2+z^2 \le 1,\ x \ge 0,\ 0 \le z \le \sqrt{2}(x^2+y^2) \}$



$z = \sqrt{2}\rho^2 \qquad z^2 = 2\rho^4$

$z^2+\rho^2 = 1$

$z>0$

$2\rho^4+\rho^2-1=0$

$t=\rho^2 \qquad 2t^2+t-1=0$

$t_{12} = \frac{-1\pm\sqrt{1+8}}{4} = \frac{-1\pm3}{4} \begin{cases} -1 \\ \frac{1}{2} \end{cases}$

$\boxed{\rho = \frac{\sqrt{2}}{2}} \qquad z = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \quad \boxed{z = \frac{\sqrt{2}}{2}}$

COORD. CILINDRICHE

$0 \le \theta \le \pi \qquad z^2+\rho^2 \le 1 \qquad 0 \le z \le \sqrt{2}\rho^2$

$INT = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta\, d\theta \iint_{D_1} \frac{z\rho}{\rho^2}\, \rho\, d\rho\, dz =$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta\, d\theta = \left[\sin\theta \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2$

$\iint_{D_1} z\, d\rho\, dz = \int_0^{\sqrt{2}/2} z\, dz \int_{\frac{\sqrt{z}}{\sqrt[4]{2}}}^{\sqrt{1-z^2}} d\rho = \int_0^{\sqrt{2}/2} z \left(\sqrt{1-z^2} - \sqrt{z}/\sqrt[4]{2} \right) dz =$

$\int_0^{\sqrt{2}/2} z\sqrt{1-z^2}\, dz - \frac{1}{2^{1/4}} \int_0^{\sqrt{2}/2} z^{\frac{3}{2}}\, dz = \frac{1}{2}\int_0^{1/2} \sqrt{1-s}\, ds - \frac{1}{2^{1/4}} \left[\frac{2}{5} z^{\frac{5}{2}} \right]_0^{\frac{\sqrt{2}}{2}}$

$= \frac{1}{2}\left[-\frac{2}{3}(1-s)^{\frac{3}{2}} \right]_0^{1/2} - \frac{2^{\frac{3}{4}}}{5} \left(2^{-1/2} \right)^{5/2} = \frac{1}{3}\left(1 - \left(\frac{1}{2}\right)^{\frac{3}{2}} \right) - \frac{2^{\frac{3}{4}}}{5} 2^{-\frac{5}{4}} =$

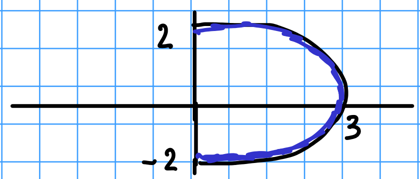
$\frac{1}{3}\left(1 - 2^{-\frac{3}{2}} \right) - \frac{1}{5} 2^{-\frac{1}{2}} = \frac{1}{3} - \frac{1}{6}\frac{\sqrt{2}}{2} - \frac{1}{5}\frac{\sqrt{2}}{2} = \frac{1}{3} - \frac{11}{30}\frac{\sqrt{2}}{2}$

FORSE...

$f(x,y) = 3x^2 + 9y^2$

$D = \{4x^2 + 9y^2 = 36,\ x \geq 0\}$

max/min f su D



PTI STAZ. VINCOLATI

$$\nabla f = \begin{pmatrix} 6x \\ 18y \end{pmatrix}$$

1 MOLTIPLICATORE

$$\begin{cases} 6x = 8\lambda x \\ 18y = 18\lambda y \\ 4x^2 + 9y^2 = 36 \quad x>0 \end{cases}$$

$\lambda \neq 0$ se no $x=y=0$

I° riga $\rightarrow$ $\cancel{x=0}$ / $6 = 8\lambda \iff \lambda = \frac{3}{4}$

$\lambda = \frac{3}{4}$ $\qquad y = \frac{3}{4} y$ $\qquad y = 0$ $\qquad \boxed{(3,0)}$

Devo aggiungere $\boxed{(0, \pm 2)}$

se invece $f = \underline{3x^2 + 9y}$ $\qquad \nabla f = \begin{pmatrix} 6x \\ 9 \end{pmatrix}$

$$\begin{cases} 6x = 8\lambda x \\ 9 = 18\lambda y \\ 4x^2 + 9y^2 = 36,\ x>0 \end{cases}$$

$\lambda = \frac{3}{4}$

$y = \frac{1}{2\lambda} = \frac{2}{3}$ $\qquad x =$

$4x^2 + 9\frac{4}{9} = 36$ $\qquad 4x^2 = 32$

$x^2 = 8$ $\qquad x = +\sqrt{2}\,2$

$\boxed{\left(2\sqrt{2}, \frac{2}{3}\right)}$ $+$ $\boxed{(0, \pm 2)}$

$$\iiint_D \frac{dx\,dy\,dz}{x^2+y^2+z^2} \qquad x^2+y^2 \le 1$$

$$\int_{-\infty}^{+\infty} dz \iint_{x^2+y^2\le 1} \frac{dx\,dy}{x^2+y^2+z^2} \overset{\text{COOR CIL.}}{=} \boxed{\int_{-\infty}^{+\infty} dz \int_0^{2\pi} d\theta \int_0^1 \frac{\rho\, d\rho}{\rho^2+z^2}} =$$

$$2\pi \int_0^1 d\rho \int_{-\infty}^{+\infty} \frac{\rho}{\rho^2+z^2}\, dz = \qquad z=\rho t \qquad dz=\rho\, dt$$

$$2\pi \int_0^1 d\rho \int_{-\infty}^{+\infty} \frac{\rho^2\, dt}{\rho^2(1+t^2)} = 2\pi \int_0^1 d\rho \left[\arctan t\right]_{-\infty}^{+\infty} = 2\pi^2 \int_0^1 d\rho = 2\pi^2$$

Nell'altro verso

$$2\pi \int_{-\infty}^{+\infty} dz \int_0^1 \frac{\rho}{\rho^2+z^2}\, d\rho = 2\pi \int_{-\infty}^{+\infty} dz \left[\frac{1}{2}\ln(\rho^2+z^2)\right]_{\rho=0}^{\rho=1} =$$

$$\pi \int_{-\infty}^{+\infty} \ln\left(\frac{1+z^2}{z^2}\right) dz = 2\pi \int_0^{+\infty} \ln\left(\frac{1+z^2}{z^2}\right) dz$$

$$\underbrace{2\pi\left[z \ln\left(1+\frac{1}{z^2}\right)\right]_0^{+\infty}}_{0} - 2\pi \int_0^{+\infty} z\, \frac{z^2}{1+z^2}\, \frac{\overset{-2z}{\cancel{2z\cdot z^2}-(1+z^2)2z}}{z^4}\, dz =$$

$$+\,2\pi \int_0^{+\infty} \frac{2\, dz}{(1+z^2)} = 4\pi\left[\arctan z\right]_0^{\infty} = 2\pi^2$$