Claudio Saccon (*)
Ingegneria Aerospaziale
Lezioni di Analisi Matematica 2 e Complementi

Lezione 72 01/06/21

email: claudio.sacconCHIOCCIOLAunipi.it
web: http://pagine.dm.unipi.it/csblog1/

Ricevimento su appuntamento da concordare per email

$$A = \left[\begin{array}{cccc|c} 1 & -1 & 1 & 0 & 0 \\ 2 & 3 & 3 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 4 & 2 & 8 & -1 & 0 \\ \hline 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

- se calcoliamo il pol. caratteristico si vede che A ha solo due autovalori: $\lambda = -1$ SEMPLICE e $\lambda = 1$ di $m_A = 4$
  $\left(p(\lambda) = -(\lambda - 1)^4 (\lambda + 1) \right)$
- Cerco la parte della matrice di Jordan relativa a $\lambda = 1$
  Pongo $= B = A - I$ (ricordo che mi serve il k: $\dim\left(\operatorname{Ker} B^k\right) = 4 = m_A$

$$B = \left[\begin{array}{cccc|c} 0 & -1 & 1 & 0 & 0 \\ 2 & 2 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 8 & -2 & 0 \\ \hline 0 & 0 & 0 & 0 & -2 \end{array}\right]$$

che rango ha B ?!

$\mathrm{I}^\circ + \mathrm{II}^\circ = \frac{1}{2}\,\mathrm{IV}^\circ \Rightarrow$ ha rango 3

$$B^2 = \left[\begin{array}{cccc|c} 0 & -1 & 1 & 0 & 0 \\ 2 & 2 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 8 & -2 & 0 \\ \hline 0 & 0 & 0 & 0 & -2 \end{array}\right] \left[\begin{array}{cccc|c} 0 & -1 & 1 & 0 & 0 \\ 2 & 2 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 8 & -2 & 0 \\ \hline 0 & 0 & 0 & 0 & -2 \end{array}\right] =$$

$$\begin{bmatrix} -2 & -2 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -4 & -4 & -6 & 2 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

← ha rango 2 (solo due righe indip. – la I° e la V°)

$$B^3 = \left[\begin{array}{c|c} \boxed{0} & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ 0 \; \cdots & -8 \end{array}\right]$$ ← ha rango 1 $\Rightarrow$ $B^k = \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & (-2)^k \end{array}\right]$

$m_0 = 0$

DUNQUE

$m_1 \quad \dim \operatorname{Ker}(B) = 2 \qquad (= n_1)$

$m_2 \quad \dim \operatorname{Ker}(B^2) = 3$

$m_3 \quad \dim \operatorname{Ker}(B^2) = 4$

se $n_i = m_i - m_{i-1}$

$n_1 = 2$

$n_2 = 1$

$n_3 = 1$

$$\operatorname{Ker} B^3 \left\{ \begin{array}{l} e_4 \\ \downarrow \\ \operatorname{Ker} B^2 \left\{ \begin{array}{l} e_3 \\ \downarrow \\ e_2 \\ \downarrow \\ 0 \end{array} \right. \end{array} \right. \qquad \begin{array}{c} e_1 \\ \downarrow B \\ 0 \end{array} \leftarrow \operatorname{Ker} B$$

TROVO e_4 cercando un vettore in $\operatorname{Ker} B^3 \setminus \operatorname{Ker} B^2$

$$e_4 = \begin{bmatrix} x \\ y \\ z \\ t \\ w \end{bmatrix} \qquad e_4 \in \operatorname{Ker} B^3 \iff w = 0$$

(x y z t sono libere)

se $B^2 e_4 \neq 0$ devo scegliere x y z t in modo che ciò avvenga per esempio $e_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow B^2 e_4 = \begin{bmatrix} -2 \\ 0 \\ 0 \\ -4 \\ 0 \end{bmatrix} = e_2$

$B e_4 = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -4 \\ 0 \end{bmatrix} = e_3$ $\left(e_4 \xrightarrow{B} e_3 \xrightarrow{B} e_2 \xrightarrow{B} 0 \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \right)$

Mi manca e_1 nel $\ker(B)$, indipendente da e_2

Vediamo come è fatto $\ker B$ $e = \begin{pmatrix} x \\ y \\ z \\ t \\ 0 \end{pmatrix}$ ca

$$\begin{cases} -y + z = 0 \\ 2x + 2y + 3z - t = 0 \\ 4x + 2y + 8z - 2t = 0 \end{cases} \qquad \begin{cases} y = z \\ 2x + 5z = t \\ \cancel{4x + 10z = 2t} \end{cases}$$

$e = \begin{bmatrix} x \\ z \\ z \\ 2x + 5z \\ 0 \end{bmatrix}$ $e_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 5 \\ 0 \end{bmatrix}$ $e_2 = \begin{bmatrix} -2 \\ 0 \\ 0 \\ -4 \\ 0 \end{bmatrix}$

$(z = 1 \quad x = 0)$ $(z = 0 \quad x = -2)$

$\Rightarrow$ Ho TROVATO $M = \begin{bmatrix} 0 & -2 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 5 & -4 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

e_5 : $A e_5 = -e_5$

$$J = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \qquad A = M J M^{-1}$$

$$e^{tJ} = \begin{bmatrix} e^{t} & 0 & 0 & 0 & 0 \\ 0 & e^{t} & te^{t} & \frac{t^2}{2}e^{t} & 0 \\ 0 & 0 & e^{t} & te^{t} & 0 \\ 0 & 0 & 0 & e^{t} & 0 \\ 0 & 0 & 0 & 0 & e^{-t} \end{bmatrix}$$

$$\begin{cases} x' = -3x + 7z \\ y' = 4x + 2y - 5z \\ z' = -2x + 5z \end{cases}$$

$$A = \begin{bmatrix} -3 & 0 & 7 \\ 4 & 2 & -5 \\ -2 & 0 & 5 \end{bmatrix}$$

$$P(\lambda) = \det \begin{bmatrix} -3-\lambda & 0 & 7 \\ 4 & 2-\lambda & -5 \\ -2 & 0 & 5-\lambda \end{bmatrix} = (2-\lambda)\det\begin{bmatrix} -3-\lambda & 7 \\ -2 & 5-\lambda \end{bmatrix}$$

$$(2-\lambda)\Big((3+\lambda)(\lambda-5)+14\Big) = (2-\lambda)\underbrace{(\lambda^2-2\lambda-1)}$$

$$\lambda_{1,2} = 1 \pm \sqrt{1+1}$$

Ho TRE AUTOVALORI DISTINTI $\lambda_1 = 1+\sqrt{2}$ $\lambda_2 = 1-\sqrt{2}$, $\lambda_3 = 2$

$$\Rightarrow \quad J = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \lambda_3 \end{bmatrix}$$

Per trovare M devo trovare 3 autovettori

$\lambda = 1+\sqrt{2}$ $\qquad e_1 \in \operatorname{Ker} \begin{bmatrix} -4-\sqrt{2} & 0 & 7 \\ 4 & 1-\sqrt{2} & -5 \\ -2 & 0 & 4-\sqrt{2} \end{bmatrix}$

$$\begin{cases} -(4+\sqrt{2})x + 7z = 0 \\ 4x + (1-\sqrt{2})y - 5z = 0 \\ \cancel{-2x \quad 0 \quad (4-\sqrt{2})z = 0} \end{cases} \qquad \det \begin{bmatrix} -4-\sqrt{2} & 7 \\ -4 & 4-\sqrt{2} \end{bmatrix} = 2-16+14 = 0$$

Posso mettere $y = 1$ e risolvere $\begin{cases} -(4+\sqrt{2})x + 7z = 0 \\ 4x - 5z = \sqrt{2}-1 \end{cases}$

....

ALLA FINE TROVO e_1, e_2, e_3 e costruisco $M = [e_1 | e_2 | e_3]$

$$A = \begin{bmatrix} 2 & -2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad \begin{cases} x' = 2x - 2y \\ y' = 2x \\ z' = -z \end{cases}$$

$$p(\lambda) = (-\lambda-1)\det \begin{bmatrix} 2-\lambda & -2 \\ 2 & -\lambda \end{bmatrix} = -(\lambda+1)\left(-2\lambda + \lambda^2 + 4\right) =$$

$$-(\lambda+1)(\underbrace{\lambda^2 - 2\lambda + 4}_{1 \pm \sqrt{1-4}}) \qquad = \boxed{1 \pm \sqrt{3}\, i}$$

complesse coniugate

$\Rightarrow$ TRE AUTOV. DISTINTI $\lambda_1 = 1+\sqrt{3}i$ $\lambda_2 = 1-\sqrt{3}i$ $\lambda_3 = -1$

$$\Rightarrow J = \begin{bmatrix} 1+\sqrt{3}i & 0 & 0 \\ 0 & 1-\sqrt{3}i & 0 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow e^{tJ} = \begin{bmatrix} e^t e^{\sqrt{3}it} & 0 & 0 \\ 0 & e^t e^{-\sqrt{3}it} & 0 \\ 0 & 0 & e^{-t} \end{bmatrix}$$

Si può fare tutto come nel caso reale: Cerco gli autovettori

- $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $(\lambda = -1)$

- $\lambda = \lambda_1 = 1+\sqrt{3}i$

$$e_1 = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} : \quad \begin{matrix} (1-\sqrt{3}i)x - 2y = 0 \\ 2x + (-1+\sqrt{3}i)y \end{matrix} \qquad y = \frac{1-\sqrt{3}i}{2} \quad x = 1$$

$$e_1 = \begin{bmatrix} 1 \\ \frac{1-\sqrt{3}i}{2} \\ 0 \end{bmatrix}$$

- Su $\lambda = \lambda_2 = \overline{\lambda_1}$ stessi calcoli

$$e_2 = \begin{bmatrix} 1 \\ 1+\sqrt{3}i \\ 0 \end{bmatrix} \Rightarrow M = \begin{bmatrix} \overbrace{\boxed{\begin{matrix} 1 & 1 \\ \frac{1-\sqrt{3}i}{2} & \frac{1+\sqrt{3}i}{2} \end{matrix}}}^{M'} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1 \end{bmatrix}$$

$$\det M' = \frac{1+\sqrt{3}i}{2} + \frac{\sqrt{3}i-1}{2} = \sqrt{3}i$$

$$M^{-1} = \begin{bmatrix} \frac{1}{\sqrt{3}i} \boxed{\begin{matrix} \frac{1+\sqrt{3}i}{2} & -1 \\ \frac{\sqrt{3}i-1}{2} & 1 \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}-i}{2\sqrt{3}} & i & 0 \\ \frac{\sqrt{3}+i}{2\sqrt{3}} & -i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$-\frac{\sqrt{3}}{3}i$

Risolviamo il sistema con condizioni $x_0 = 1$ $y_0 = 0$ $z_0 = -1$

$$Y(t) = M e^{tJ} M^{-1} Y_0 = \qquad Y_0 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$M e^{tJ} \begin{bmatrix} \frac{\sqrt{3}-i}{2\sqrt{3}} \\ \frac{\sqrt{3}+i}{2\sqrt{3}} \\ 1 \end{bmatrix} =$$

$$M \begin{bmatrix} e^{t} e^{\sqrt{3}it} & 0 & 0 \\ 0 & e^{t} e^{-\sqrt{3}it} & 0 \\ 0 & 0 & e^{-t} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}-i}{2\sqrt{3}} \\ \frac{\sqrt{3}+i}{2\sqrt{3}} \\ 1 \end{bmatrix} =$$

$$M \begin{bmatrix} \frac{\sqrt{3}-i}{2\sqrt{3}} e^{t} e^{\sqrt{3}it} \\ \frac{\sqrt{3}+i}{2\sqrt{3}} e^{t} e^{\sqrt{3}it} \\ e^{-t} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ \frac{1-\sqrt{3}i}{2} & \frac{1+\sqrt{3}i}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}-i}{2\sqrt{3}} e^{t} e^{\sqrt{3}it} \\ \frac{\sqrt{3}+i}{2\sqrt{3}} e^{t} e^{-\sqrt{3}it} \\ e^{-t} \end{bmatrix}$$

CONIUGATI

$$= \begin{bmatrix} \frac{\sqrt{3}-i}{2\sqrt{3}} e^{t} e^{\sqrt{3}it} + \frac{\sqrt{3}+i}{2\sqrt{3}} e^{t} e^{-\sqrt{3}it} \\ \frac{1-\sqrt{3}i}{2} \, \frac{\sqrt{3}-i}{2\sqrt{3}} e^{t} e^{\sqrt{3}it} + \frac{1+\sqrt{3}i}{2} \, \frac{\sqrt{3}+i}{2} e^{t} e^{-\sqrt{3}it} \\ e^{-t} \end{bmatrix}$$

CONIUGATI

$$= 2 \, \mathrm{Re} \begin{bmatrix} \left(\frac{\sqrt{3}-i}{2\sqrt{3}}\right) e^{t} \left(\cos(\sqrt{3}t) + i \sin(\sqrt{3}t)\right) \\ -\frac{i}{\sqrt{3}} e^{t} \left(\cos(\sqrt{3}t) + i \sin(\sqrt{3}t)\right) \\ e^{-t} \end{bmatrix} =$$

$$\left(\frac{1-\sqrt{3}i}{2} \, \frac{\sqrt{3}-i}{2\sqrt{3}} = \frac{\sqrt{3} - i - 3i - \sqrt{3}}{4\sqrt{3}} = \frac{-i}{\sqrt{3}} \right)$$

$$= 2\,\mathrm{Re}\begin{bmatrix} e^{t}\left(\frac{\sqrt{3}}{2\sqrt{3}}\cos(\sqrt{3}t) + \frac{1}{2\sqrt{3}}\sin(\sqrt{3}t)\right) + i(\,-\,-\,-\,) \\ \frac{1}{\sqrt{3}}e^{t}\sin(\sqrt{3}t) + i\,\cdot\cdot \\ e^{-t} \end{bmatrix} =$$

$$Y(t) = \begin{bmatrix} e^{t}\left(\cos(\sqrt{3}t) + \frac{1}{\sqrt{3}}\sin(\sqrt{3}t)\right) \\ e^{t}\,\frac{\sin(\sqrt{3}t)}{\sqrt{3}} \\ e^{-t} \end{bmatrix} \qquad \left(Y(0) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right)$$

$$A = \begin{bmatrix} 4 & -1 & -1 \\ -2 & 2 & 1 \\ 4 & -2 & 0 \end{bmatrix} \qquad B(t) = \begin{bmatrix} 0 \\ 0 \\ e^{t} \end{bmatrix}$$

$$(P) = \begin{cases} x' = 4x - y - z \\ y' = -2x + 2y + z \\ z' = 4x - 2y + e^{t} \end{cases}$$

$$P(\lambda) = \det\begin{bmatrix} 4-\lambda & -1 & -1 \\ -2 & 2-\lambda & 1 \\ 4 & -2 & -\lambda \end{bmatrix} = (4-\lambda)(2-\lambda)(-\lambda) - 4 - 4 + $$
$$- \left[(4-\lambda)(-2)1 + (2-\lambda)4(-1) + (-\lambda)(-2)(-1) \right]$$

$$= -\lambda(\lambda^2 - 6\lambda + 8) - 8 - \left[-8 + 2\lambda - 8 + 4\lambda - 2\lambda \right] =$$

$$-\lambda^3 + 6\lambda^2 - 8\lambda - 8 + 16 - 4\lambda = -\lambda^3 + 6\lambda^2 - 12\lambda + 8$$

$$= (2-\lambda)^3$$ UN SOLO AUTOVALORE $\boxed{\lambda = 2}$ $m_A = 3$

$$B = A - 2I = \begin{bmatrix} 2 & -1 & -1 \\ -2 & 0 & 1 \\ 4 & -2 & -2 \end{bmatrix} \leftarrow \text{ha rango } 2 \,!$$

Ker ha dim (1)

$m_G = 1 \implies$ UNA SEQUENZA $e_3 \to e_2 \to e_1 \to 0$

$$B^2 = \dots = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 0 & 0 \\ 4 & 0 & -2 \end{bmatrix} \qquad B^3 = 0 \quad \left(\dim \operatorname{Ker} B^3 = 3\right)$$

$\dim \operatorname{Ker} B^2 = 2$

$$J = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

Prendo e_3 con $B^2 e_3 \neq 0$

e poi $e_1 = B^2 e_3$ $\quad e_2 = B e_3$

per esempio

$$e_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad e_1 = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix} \qquad e_2 = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$$

OPPURE

$$e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad e_1 = \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} \qquad e_2 = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$$

---

(1) Si trovi una sol. di (P) della forma

$$\hat{Y}(t) = \begin{bmatrix} a \\ b \\ c \end{bmatrix} e^t \qquad (a, b, c \in \mathbb{R} \text{ da trovare})$$

Cerco a, b, c IMPONENDO che $\hat{Y}' = A\hat{Y} + B(t)$ cioè

$$e^t \begin{bmatrix} a \\ b \\ c \end{bmatrix} = e^t \begin{bmatrix} 4 & -1 & -1 \\ -2 & 2 & 1 \\ 4 & -2 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} + e^t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \iff$$

$$(A-I)\begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} 3a - b - c = 0 \\ -2a + b + c = 0 \\ 4a - 2b - c = -1 \end{cases} \iff \begin{cases} -2a + b + c = 0 & (\mathrm{II}) \\ a = 0 & (\mathrm{I}+\mathrm{II}) \\ 2a - b = -1 & (\mathrm{II}+\mathrm{III}) \end{cases}$$

$$\boxed{a=0 \quad b=1 \quad c=-1}$$

DUNQUE $\hat{Y}(t) = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} e^t = \begin{bmatrix} 0 \\ e^t \\ -e^t \end{bmatrix}$ Nota $\hat{Y}(0) = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

(2) TROVARE LA SOL. DI (P) $Y(t)$ tale che $Y(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Conviene cercare Y della forma $\boxed{Y(t) = Y_0(t) + \hat{Y}(t)}$

dove Y_0 è sol dell'omogenea $Y_0' = AY_0$

LA CONDIZIONE $Y(0) = \vec{0} \iff Y_0(0) + \hat{Y}(0) = \vec{0}$

$\iff Y_0(0) = -\hat{Y}(0) = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$!!

Dunque Y_0 deve verificare

$$\boxed{\begin{cases} Y_0' = AY_0 \\ Y_0(0) = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \end{cases}}$$ ← lo cerco col solito sistema $Y_0(t) = e^{tA}\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

Vediamo se posso prendere $e_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ (Tornando alla forma di Jordan)

$B^2 e_3 \neq 0$ $\underline{\text{sì}}$ $B^2 e_3 = \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} = e_1$

$Be_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$M = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & -1 \\ -2 & 0 & 1 \end{bmatrix} \qquad J = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \qquad e^{tJ} = e^{2t}\begin{bmatrix} 1 & t & t^2/2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow Y_0(t) = M \cdot (e^{2t}) \begin{bmatrix} 1 & t & t^2/2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{M^{-1}\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \vec{e}_3} = e^{2t} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & -1 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} t^2/2 \\ t \\ 1 \end{bmatrix}$$

$$= e^{2t} \begin{bmatrix} -t^2/2 \\ t-1 \\ -t^2+1 \end{bmatrix} = Y_0(t)$$

Alla fine

$$Y(t) = Y_0(t) + \tilde{Y}(t) = e^{2t} \begin{bmatrix} -t^2/2 \\ t-1 \\ -t^2+1 \end{bmatrix} + e^{t} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

#

$$A = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} A_1^{-1} & 0 \\ 0 & 1 \end{bmatrix}$$

$$A_1^{-1} = \frac{1}{ad-bc} \begin{bmatrix} \cdots \end{bmatrix}$$

Nella "Jordanizzazione" si trova K

tale che $\dim \operatorname{Ker} B^{K-1} < \dim \operatorname{Ker} B^{K} = m_A(\lambda)$

Potrebbe essere $K = m_A$ (solo nel caso $m_G = 1$

e c'è una sola sequenza $\underbrace{e_K \xrightarrow{B} e_{K-1} \rightarrow \cdots \xrightarrow{B} e_0}_{K}$

$$J = \begin{bmatrix} \lambda & 1 & & \\ & \ddots & \ddots & \\ & 0 & & 1 \\ & & & \lambda \end{bmatrix}$$

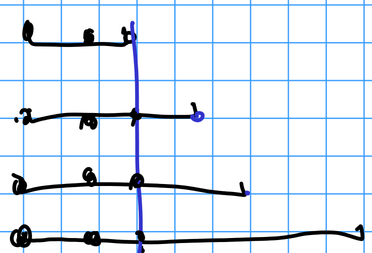
IN GENERALE $K \leq m_A$

$m_0 = \dim \operatorname{Ker} B^0 = 0$
$m_1 = \dim \operatorname{Ker} B$
$m_2 = \dim \operatorname{Ker} B^2$
$\vdots$
$m_K = \dim \operatorname{Ker} B^K = m_A$

$0 < m_1 < m_2 < \cdots < m_K = m_A$

$n_1 = m_1$
$n_2 = m_2 - m_1$
$\vdots$
$n_K = m_K - m_{K-1}$

K



3
2
1

$n_K =$

$n_i \leq n_{i-1}$

n_2 elementi

n_1 elementi (autovettori)

10×10 $m_A = 10$ $m_G = 3$

$$\boxed{m_1 = 3 \quad m_2 = 5 \quad m_3 = 7 \quad m_4 = 9 \quad m_5 = 10}$$

$n_1 = 3$ $n_2 = 2$ $n_3 = 2$ $n_4 = 2$ $n_5 = 1$

- c'è un solo elemento e_{10} in $\underbrace{\operatorname{Ker} B^5}_{= \mathbb{R}^{10}} \setminus \operatorname{Ker} B^4$

$$e_{10} \xrightarrow{B} e_9 \xrightarrow{B} e_8 \xrightarrow{B} e_7 \xrightarrow{B} e_6 \xrightarrow{B} 0$$

• $\dim \mathrm{Ker}\, B^4 = 9$ $\dim \mathrm{Ker}\, B^3 = 7$ – lo scarto è 2

ho già e_9 in $\mathrm{Ker}\, B^4 \setminus \mathrm{Ker}\, B^3$ trovo un altro elemento e_5

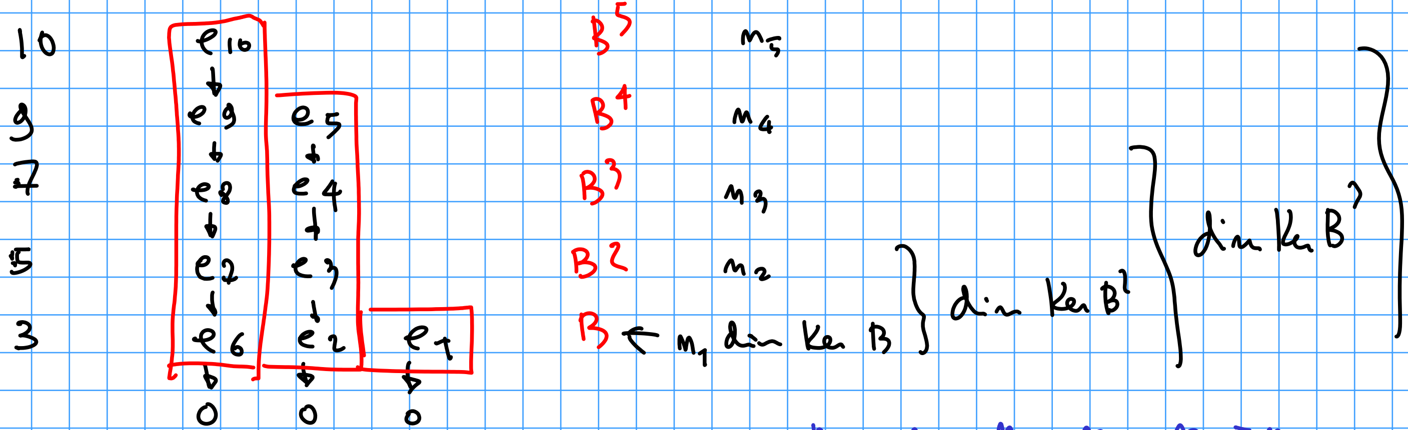
da e_5 costruisco $e_5 \xrightarrow{B} e_4 \xrightarrow{B} e_3 \xrightarrow{B} e_2 \xrightarrow{B} 0$

• $\dim \mathrm{Ker}\, B^3 = 7$ $\dim \mathrm{Ker}\, B^2 = 5$ scarto $= 2$

ho già e_8 e e_4 non c'è niente di nuovo

• $\dim \mathrm{Ker}\, B^2 = 5$ $\dim \mathrm{Ker}\, B = 3$

ho già e_7 ed e_3 trovo e_{10} in $\mathrm{Ker}\, B$



m_5	m_4	m_3	m_2	$m_1 = m_1$
1	2	2	2	3

$$\boxed{J_5 \quad J_4 \quad J_1}$$

$$\begin{cases} \delta_5 = m_5 & \text{un } 5\text{-blocco} \\ \delta_4 = m_4 - m_5 = 1 & \text{un } 4\text{-blocco} \\ \delta_3 = m_3 - m_4 = 0 & 0 \ \ 3 \text{ blocchi} \\ \delta_2 = m_2 - m_3 = 0 & 0 \ \ 3 \text{ blocchi} \\ \delta_1 = m_1 - m_2 = 1 & 1 \ \ 1\text{-blocco} \end{cases}$$