

# Esercizi su limiti di funzione

$$1) \lim_{x \rightarrow 0} \frac{3^x - 1}{x}$$

$$4) \lim_{x \rightarrow 0} \frac{\arcsin(2x)}{\sin(3x)}$$

$$7) \lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^{x^2}$$

$$10) \lim_{x \rightarrow 2} \frac{e^x - e^2}{2x - 4}$$

$$12) \lim_{x \rightarrow 0^+} \frac{\sqrt{\cos(\sqrt{x})} - e^x}{\tan(x)}$$

$$14) \lim_{x \rightarrow 0} \frac{\sqrt{4 - x^2} - 2 \cos(x)}{x^2}$$

$$16) \lim_{x \rightarrow +\infty} x 2^x \left[ \left( \frac{1 + 2^x}{2^x} \right)^{\frac{1}{x}} - 1 \right]$$

$$2) \lim_{x \rightarrow 0} \frac{\sin(x^2)}{\sin^2(x)}$$

$$5) \lim_{x \rightarrow 0} (1 + 3x)^{\frac{1}{x}}$$

$$8) \lim_{x \rightarrow +\infty} \left( \frac{x^2 + 1}{x^2 - 1} \right)^{\frac{x^4 + 1}{x^2 - 1}}$$

$$11) \lim_{x \rightarrow 1} \frac{\sqrt{x} - \cos(x - 1)}{x - 1}$$

$$13) \lim_{x \rightarrow 0} \frac{\sqrt{\cos(x)} - e^x}{\sin(x)}$$

$$15) \lim_{x \rightarrow +\infty} 2^x \left[ \left( \frac{1 + 2^x}{2^x} \right)^{\frac{1}{x}} - 1 \right]$$

$$17) \lim_{x \rightarrow \frac{\pi}{2}} (2x - \pi) \tan(x)$$

$$3) \lim_{x \rightarrow 0} \frac{\sin(x)}{x}$$

$$6) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$$

$$9) \lim_{x \rightarrow 1} \frac{\ln(x)}{x - 1}$$

$$18) \lim_{x \rightarrow +\infty} x (\ln(x^2 + 2x + 2) - \ln(x^2 - 3x + 1))$$

$$19) \lim_{x \rightarrow +\infty} x^2 (\ln(1 + 2^x) - \ln(3 + 2^x))$$

$$20) \lim_{x \rightarrow +\infty} 2^x (\ln(1 + 2^x) - \ln(3 + 2^x))$$

$$21) \lim_{x \rightarrow +\infty} 2^x (\ln(4 + 2^x + 3^x) - \ln(1 - 2^x + 3^x))$$

$$22) \lim_{x \rightarrow +\infty} 2^x (\ln(4 + 2^x + 3^x) - \ln(1 + 2^x + 3^x))$$

$$23) \lim_{x \rightarrow +\infty} 2^x (\ln(4 + 2^x + 4^x) - \ln(1 - 2^x + 4^x))$$

$$24) \lim_{x \rightarrow +\infty} x - \ln(1 + 2^x + x^2)$$

$$25) \lim_{x \rightarrow +\infty} x - \ln(1 + e^x + x^2)$$

$$26) \lim_{x \rightarrow +\infty} e^x (x - \ln(1 + e^x + x^2))$$

$$27) \lim_{x \rightarrow +\infty} \frac{e^x}{x^2} (x - \ln(1 + e^x + x^2))$$

$$28) \lim_{x \rightarrow 0} \frac{2\sqrt{1+x} - x - 2\cos(x)}{x \sin(x)} \quad (*)$$

$$29) \lim_{x \rightarrow 0} (2\sqrt{1+x} - 1)^{\frac{1}{x}}$$

$$30) \lim_{x \rightarrow 0} \left( \frac{2\sqrt{1+x} - 2}{x} \right)^{\frac{1}{x}} \quad (*)$$

# Risposte

(1)	$\ln(3)$	(2)	1	(3)	-2	(4)	$\frac{2}{3}$	(5)	$e^3$
(6)	1	(7)	$+\infty$	(8)	$e^2$	(9)	1	(10)	$\frac{e^2}{2}$
(11)	$\frac{1}{2}$	(12)	$-\frac{5}{4}$	(13)	-1	(14)	$\frac{3}{4}$	(15)	0
(16)	1	(17)	-2	(18)	5	(19)	0	(20)	-2
(21)	$+\infty$	(22)	0	(23)	2	(24)	$+\infty$	(25)	0
(26)	$+\infty$	(27)	1	(28)	$\frac{3}{4}$	(29)	$e$	(30)	$\frac{1}{\sqrt[4]{e}}$

Per i limiti contrassegnati da (\*) serve il limite

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1 - x/2}{x^2} = -\frac{1}{8} \Leftrightarrow \sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + o(x^2)$$

che si fa “razionalizzando” due volte.