

$$F(x) = \int_1^x t \lg^2 t \cdot t \, dt$$

$$F(x) = \int_1^x \delta \lg^2 \delta - \delta \, d\delta$$

$$\text{DOMINIO: } \delta > 0 \quad x \in (0, +\infty) \quad F(1) = 0$$

$$\lim_{x \rightarrow +\infty} F(x) = +\infty$$

$$\lim_{x \rightarrow 0^+} F(x) = 1/5$$

$$\begin{aligned} \int_0^1 \delta \lg^2 \delta - \delta \, d\delta &= \left[\frac{\delta^2}{2} \lg^2 \delta \right]_0^1 - \int_0^1 \delta \lg \delta \, d\delta - \int_0^1 \delta \, d\delta = \\ &= - \left[\frac{\delta^2}{2} \lg \delta \right]_0^1 + \int_0^1 \frac{\delta}{2} \, d\delta - \left[\delta^2/2 \right]_0^1 = \left[\delta^2/4 \right]_0^1 - 1/2 = 1/4 - 1/2 = -1/4 \end{aligned}$$

$$\begin{aligned} F'(x) &= x \lg^2 x - x = x (\lg^2 x - 1) = \\ &= x (\lg x - 1)(\lg x + 1) \end{aligned}$$

$$\begin{array}{ccccccc} 0 & 1/e & e \\ \odot & + & + & + & \bullet & - & - & - & - & \bullet & + & + & + & + & + \end{array}$$

$$F''(x) = \lg^2 x + 2 \lg x - 1$$

$$\begin{array}{ccccccc} & & \sqrt{2}-1 \\ & & e \\ \bullet & - & - & - & \bullet & + & + & + & + & + & + \end{array}$$

$$e^2 + 2e - 1 = 0 \quad e = \frac{-2 \pm \sqrt{5+5}}{2} < \begin{array}{l} -1 - \sqrt{2} \\ -1 + \sqrt{2} \end{array}$$

$$\leadsto \lg x = \sqrt{2} - 1 \quad x = e^{\sqrt{2}-1}$$

