

$$\begin{cases} zw^2 = |z|^2 |w| \\ w^2 + 3\bar{z} = 4i \end{cases}$$

$$\begin{cases} \bar{z} w^2 = |\bar{z}|^2 |w| \leadsto \bar{z} (w^2 - \bar{z} |w|) = 0 \\ w^2 + 3\bar{z} = 4i \end{cases} \begin{cases} \bar{z} = 0 \\ w^2 - \bar{z} |w| = 0 \end{cases}$$

$$\underline{\bar{z} = 0}$$

$$w^2 + 3\bar{z} = 4i \leadsto w^2 = 4i = 4e^{\frac{\pi}{2}i}$$

$$\leadsto \begin{cases} w_1 = 2e^{\frac{\pi}{4}i} = \sqrt{2}(1+i) \\ w_2 = 2e^{\frac{3\pi}{4}i} = \sqrt{2}(-1-i) \end{cases}$$

$$\underline{w^2 - \bar{z} |w| = 0}$$

$$\begin{cases} w^2 + 3\bar{z} = 4i \\ \bar{z} = \frac{w^2}{|w|}, \quad |w| \neq 0 \end{cases} \leadsto w^2 + 3 \frac{w^2}{|w|} = 4i$$

$$w^2 \bar{w}^2 + 3 \frac{w^2}{|w|} \bar{w}^2 = 4i \bar{w}^2 \leadsto |w|^4 + 3|w|^3 = 4i \bar{w}^2$$

$$\underline{\text{OSS}} \quad \text{LHS} \in \mathbb{R}^+ \leadsto \bar{w}^2 \in \mathbb{C} \text{ con } \text{Im}(\bar{w}^2) \in (-\infty, 0)$$

$$\text{PONIAMO: } \bar{w}^2 = -b i, \quad b \in \mathbb{R}^+$$

$$\leadsto b^2 + 3b\sqrt{b} = 4b \leadsto b + 3\sqrt{b} - 4 = 0$$

$$\sqrt{b} = \frac{-3 \pm \sqrt{9+16}}{2} = \begin{cases} 1 \\ -4 \end{cases} \leadsto b = 1 \quad \text{ASSUNDO}$$

$$\leadsto \bar{w}^2 = -i = e^{-\frac{\pi}{2}i} \leadsto \begin{cases} \bar{w}_1 = \frac{-1+i}{\sqrt{2}} \\ \bar{w}_2 = \frac{1-i}{\sqrt{2}} \end{cases} \leadsto \begin{cases} w_1 = \frac{-1-i}{\sqrt{2}} \\ w_2 = \frac{1+i}{\sqrt{2}} \end{cases}$$

$$\begin{cases} |w_2| = |w_2| = 1 \\ w_2^2 = \overline{w_2}^2 = 1 \end{cases} \leadsto \overline{z} = \frac{w^2}{|w|} = 1 \leadsto z = -1$$

RESTA IL CASO $|w|=0$ $\Leftrightarrow w=0$

$$\begin{cases} \cancel{z w^2 = |z|^2 |w|} \\ \cancel{w^2 + 3\overline{z}} = 5 \end{cases} \leadsto 3\overline{z} = 5 \leadsto z = -\frac{5}{3}$$