BY GIMUSI 25/07/17

$$\int_0^{+\infty} \frac{\sin x - \sin^2 x}{x^e + \cos x}\,dx = \int_0^{1} \frac{\sin x - \sin^2 x}{x^e + \cos x}\,dx + \int_1^{+\infty} \frac{\sin x - \sin^2 x}{x^e + \cos x}\,dx$$

a) STUDIAMO $\displaystyle\int_0^{1} \frac{\sin x - \sin^2 x}{x^e + \cos x}\,dx$

BRUTAL MODE

$$x \to 0 \quad \begin{cases} \sin x - \sin^2 x \sim x \\ x^{e} + \cos x \sim x^{e+1} \end{cases}$$

$$\frac{\sin x - \sin^2 x}{x^e + \cos x} \sim \frac{x}{x^{e+1}} = \frac{1}{x^e} \quad \rightsquigarrow \text{DIVERGE}$$

RIGOROSO

$\sin x - \sin^2 x \geq 0 \quad x \in [0,1] \rightsquigarrow$ CONFRONTO OK

$$\int_0^{1} \frac{\sin x - \sin^2 x}{x^e + \cos x}\,dx \geq \int_0^{1} \frac{\sin x - \sin^2 x}{x^{e+1}}\,dx = +\infty$$

$\rightsquigarrow$ CONF. ASINT. CON $\frac{1}{x^e}$ PER $x \to 0$

$$\frac{\dfrac{\sin x - \sin^2 x}{x^{e+1}}}{\dfrac{1}{x^e}} = \frac{\sin x - \sin^2 x}{x} = \frac{x + o(x)}{x} \to 1$$

$\rightsquigarrow$ POSSIAMO CONCLUDERE CHE

$$\int_0^{+\infty} \frac{\sin x - \sin^2 x}{x^e + \cos x}\,dx \quad \text{DIVERGE}$$

b) STUDIAMO ANCHE $\displaystyle\int_1^{+\infty} \frac{\sin x - \sin^2 x}{x^e + \cos x}\, dx$

OSS NON NECESSARIO

CONSIDERIAMO $\displaystyle\int_1^{+\infty} \left| \frac{\sin x - \sin^2 x}{x^e + \cos x} \right| dx < +\infty$

INFATTI $\displaystyle\left| \frac{\sin x - \sin^2 x}{x^e + \cos x} \right| \leq \frac{2}{x^{e-1}}$

E $\displaystyle\int_2^{+\infty} \frac{2}{x^{e-1}}\, dx < +\infty$

$\rightsquigarrow$ PER CRITERIO ASS. INTEGRABILITÀ

$\displaystyle\int_1^{+\infty} \frac{\sin x - \sin^2 x}{x^e + \cos x}\, dx$ CONVERGE