a) $$\int_1^{+\infty} \frac{1}{x^2}\arctan\left(\frac{x}{x+2}\right)dx$$

$$\frac{1}{x^2}\,\text{ARCTAN}\left(\frac{x}{x+2}\right) \geq 0 \qquad x \in [1, +\infty)$$

CONFR. ASINTOTICO CON $\frac{1}{x^2}$

$$\frac{\frac{1}{x^2}\,\text{ARCTAN}\left(\frac{x}{x+2}\right)}{\frac{1}{x^2}} \to \text{ARC TAN}(1) = \frac{\pi}{4}$$

$$\rightsquigarrow \int_1^{+\infty} \frac{1}{x^2}\,\text{ARCTAN}\left(\frac{x}{x+2}\right) < +\infty$$

b) $$\int_{-2}^{-1} \frac{1}{x^2}\,\text{ARCTAN}\left(\frac{x}{x+2}\right)dx$$

SOSTITUZIONE $\quad x+2 = z$

$$\int_{-2}^{-1} \frac{1}{x^2}\,\text{ARCTAN}\left(\frac{x}{x+2}\right)dx = \int_0^1 \frac{1}{(z-2)^2}\,\text{ARCTAN}\left(\frac{z-2}{z}\right)dz$$

CON $\quad \left|\frac{1}{(z-2)^2}\,\text{ARCTAN}\left(\frac{z-2}{z}\right)\right| \leq M \qquad M \in \mathbb{R}$

$$\rightsquigarrow \int_{-2}^{-1} \frac{1}{x^2}\,\text{ARCTAN}\left(\frac{x}{x+2}\right)dx \qquad \text{CONVERGE}$$

c) $$\int_0^1 \frac{1}{x^2}\arctan\left(\frac{x}{x+2}\right)dx$$

$$\frac{1}{x^2}\arctan\left(\frac{x}{x+2}\right) \geq 0 \qquad x \in (0, +\infty)$$

BRUTAL MODE

$$\frac{1}{x^2}\arctan\left(\frac{x}{x+2}\right) = \frac{1}{x^2}\,\frac{x}{x+2}\,\frac{\arctan\left(\frac{x}{x+2}\right)}{\frac{x}{x+2}} \sim \frac{1}{x^2+2x} \sim \frac{1}{x}$$

CONFR. ASINTOTICO CON $\frac{1}{x}$

$$\frac{\frac{1}{x^2}\arctan\left(\frac{x}{x+2}\right)}{\frac{1}{x}} = \frac{\arctan\left(\frac{x}{x+2}\right)}{x} =$$

$$= \frac{\arctan\left(\frac{x}{x+2}\right)}{\frac{x}{x+2}}(x+2) \rightarrow 2$$

$$\rightsquigarrow \int_0^1 \frac{1}{x^2}\arctan\left(\frac{x}{x+2}\right)dx = +\infty$$