Dato l'integrale

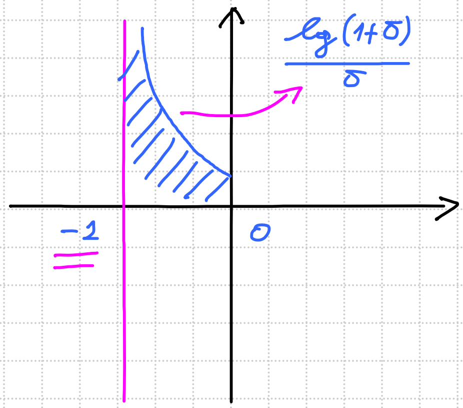
$$\int_1^x (log(1+t))/t\,dt$$

si chiede di determinare gli x reali per cui l'integrale converge.

$$f(\delta) = \frac{\log(1+\delta)}{\delta} \qquad \delta > -1 \qquad \delta \neq 0$$

$$\lim_{\delta \to 0} \frac{\log(1+\delta)}{\delta} = 1 \qquad \lim_{\delta \to -1^+} \frac{\log(1+\delta)}{\delta} = +\infty$$

$$\lim_{\delta \to +\infty} \frac{\log(1+\delta)}{\delta} = 0$$



PROBLEMI IN X = -1, CONSIDERIAMO:

$$\int_{-1}^{0} \frac{\log(1+\delta)}{\delta}\, d\delta = ?$$

CAMBIO DI VARIABILI: $1+\delta = e^{-Y}$ $\quad d\delta = -e^{-Y}dY$

$$\rightsquigarrow \int_{+\infty}^{0} \frac{-Y(-e^{-Y})}{e^{-Y}-1}\, dY = \int_{+\infty}^{0} \frac{Y}{1-e^{Y}}\, dY = \int_{0}^{+\infty} \frac{Y}{e^{Y}-1}\, dY$$

$$\lim_{Y \to +\infty} \frac{Y}{e^{Y}-1} = 0 \qquad \lim_{Y \to 0^+} \frac{Y}{e^{Y}-1} = 1$$

$$\int_{0}^{+\infty} \frac{Y}{e^{Y}}\, dY = \left[-e^{-Y}(Y+1)\right]_{0}^{+\infty} = 1$$

$$\lim_{Y \to +\infty} \frac{\cancel{Y}}{e^{Y}-1} \cdot \frac{e^{Y}}{\cancel{Y}} = 1$$

$$\Rightarrow \int_0^{+\infty} \frac{y}{e^y - 1}\, dy \quad \text{CONVERGE} \quad \text{PER C.A. CON} \quad \int_0^{+\infty} \frac{y}{e^y}\, dy$$

$$\Rightarrow \int_{-1}^{0} \frac{\lg(1+s)}{s}\, ds \quad \text{CONVERGE}$$

$$\rightsquigarrow \int_{1}^{x} \frac{\lg(1+s)}{s}\, ds \quad \text{CONVERGE} \quad \forall x \geq -1$$