BY GIMUSI 28/11/16

$$\int_1^{+\infty} \frac{x}{1-x^2}\,dx = -\infty$$

MODO 1

$$\frac{x}{1-x^2} = \frac{a}{1-x} + \frac{b}{1+x} = \frac{(a-b)x+(a+b)}{1-x^2} \qquad \begin{cases} a-b=1 \\ a+b=0 \end{cases} \qquad \begin{cases} a=1/2 \\ b=-1/2 \end{cases}$$

$$\frac{x}{1-x^2} = \frac{1/2}{1-x} - \frac{1/2}{1+x}$$

$$\int_1^{+\infty} \frac{x}{1-x^2}\,dx = \frac{1}{2}\int_1^{+\infty} \frac{1}{1-x}\,dx - \frac{1}{2}\int_1^{+\infty} \frac{1}{1+x}\,dx = -\infty$$

INFATTI:

(i) $1-x=-y \qquad dx=dy \qquad \int_1^{+\infty} \frac{1}{1-x}\,dx = -\int_0^{+\infty} \frac{1}{y}\,dy = -\infty$

(ii) $\int_1^{+\infty} \frac{1}{1+x}\,dx = +\infty$ PER C.A. CON $\int_1^{+\infty} \frac{1}{x}\,dx$

MODO 2

$$\int_1^{+\infty} \frac{x}{1-x^2}\,dx = -\int_1^{+\infty} \frac{x}{x^2-1}\,dx = -\left[\frac{1}{2}\log(x^2-1)\right]_{1+\varepsilon}^{\frac{1}{\varepsilon}} \qquad \varepsilon \to 0^+$$

$$= -\frac{1}{2}\left(\log\left(\frac{1}{\varepsilon^2}-1\right) - \log\left((1+\varepsilon)^2-1\right)\right) =$$

$$= -\frac{1}{2}\log\Bigg(\underbrace{\frac{1-\varepsilon^2}{\varepsilon^2}\,\frac{1}{\varepsilon^2+2\varepsilon}}_{\to +\infty}\Bigg) \to -\infty$$