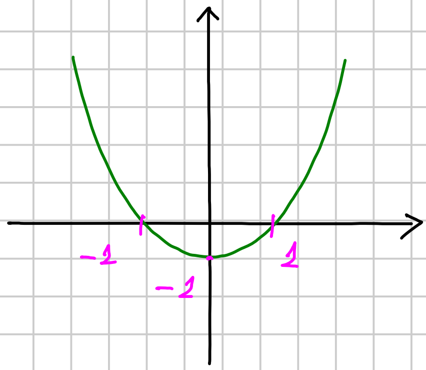
BY GIMUSI 28/07/16

$$\int_0^3 \sqrt{1+|x^2-1|}\,dx$$



$$\int_0^3 \sqrt{1+|x^2-1|}\,dx = \int_0^1 \sqrt{2-x^2}\,dx + \int_1^3 x\,dx$$

$$\int_0^1 \sqrt{2-x^2}\,dx = \qquad x=\sqrt{2}\sin\theta \qquad dx=\sqrt{2}\cos\theta\,d\theta$$

$$= \int_0^{\pi/4} \sqrt{2-2\sin^2\theta}\,(\sqrt{2}\cos\theta)\,d\theta =$$

$$= \int_0^{\pi/4} \sqrt{2}\,\sqrt{2}\,\cos^2\theta\,d\theta = 2\int_0^{\pi/4} \cos^2\theta\,d\theta =$$

$$= 2\cdot\frac{1}{2}\Big[\theta+\sin\theta\cos\theta\Big]_0^{\pi/4} = \frac{\pi}{4}+\frac{1}{2}$$

$$\int_1^3 x\,dx = \left[\frac{x^2}{2}\right]_1^3 = \frac{9}{2}-\frac{1}{2} = 4$$

$$\rightsquigarrow \int_0^3 \sqrt{1+|x^2-1|}\,dx = \int_0^1 \sqrt{2-x^2}\,dx + \int_1^3 x\,dx =$$

$$= \frac{\pi}{4}+\frac{1}{2}+4 = \frac{\pi}{4}+\frac{9}{2}$$