$$\sum_{n=0}^{\infty} \left(\sqrt{1+n^2} - n\right)^{\alpha}$$

Sia $b_n = \dfrac{1}{n^{\alpha}}$

Essendo $\displaystyle \lim_{n \to +\infty} \frac{a_n}{b_n} = \lim_{n \to +\infty} \left[\frac{\sqrt{1+n^2} - n}{1/n} \right]^{\alpha} =$

$$\lim_{n \to +\infty} \left[n\sqrt{1+n^2} - n^2 \right]^{\alpha} = \lim_{n \to +\infty} \left[n^2 \sqrt{1 + \frac{1}{n^2}} - n^2 \right]^{\alpha} =$$

$$\lim_{n \to +\infty} \left[n^2\left(1 + \frac{1}{2n^2} + o\left(\frac{1}{n^2}\right)\right) - n^2 \right]^{\alpha} =$$

$$\lim_{n \to +\infty} \left[n^2 + \frac{1}{2} - n^2 \right]^{\alpha} = \left(\frac{1}{2}\right)^{\alpha}$$

Per $\alpha > 0$ è $0 < \left(\frac{1}{2}\right)^{\alpha} < 1$; quindi, complessivamente,

la serie data si comporta come $\sum \frac{1}{n^{\alpha}}$ e converge per $\alpha > 1$.