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Si potrebbe risolvere il seguente limite di successione $((2n)!/(n!)^2)^{1/n}$, ricorrendo al confronto con l'integrale di una somma?

$$\left[\frac{(2n)!}{(n!)^2}\right]^{1/n} \to 4$$

MODO 1 – CRITERIO RAPPORTO -> RADICE

$$\boxed{\text{SIA } a_n > 0 \text{ DEFINITIVAMENTE } \quad \frac{a_{n+1}}{a_n} \to l \Rightarrow \sqrt[n]{a_n} \to l}$$

$$a_n = \frac{(2n)!}{n!^2}$$

$$\frac{a_{n+1}}{a_n} = \frac{(2n+2)!}{(n+1)!^2} \frac{n!^2}{(2n)!} = \frac{(2n+2)(2n+1)\cancel{(2n)!}}{(n+1)^2 \cdot \cancel{n!^2}} \frac{\cancel{n!^2}}{\cancel{(2n)!}} \to 4$$

MODO 2 – STIRLING

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \qquad n \to +\infty$$

$$\left[\frac{(2n)!}{(n!)^2}\right]^{1/n} \sim \left[\sqrt{4\pi n}\left(\frac{2n}{e}\right)^{2n} \frac{1}{2\pi n}\left(\frac{e}{n}\right)^{2n}\right]^{1/n} =$$

$$= \left[\frac{\sqrt{4\pi n}}{2\pi n} 2^{2n}\right]^{1/n} = 4 \cdot \underbrace{\left(\frac{1}{\sqrt{\pi n}}\right)^{1/n}}_{\to 1} \to 4$$

$$\left(\frac{1}{\sqrt{\pi n}}\right)^{1/n} = e^{\frac{1}{n}\log\left(\frac{1}{\sqrt{\pi n}}\right)} \to 1$$

MODO 3 – CONFRONTO SERIE-INTEGRALI

$$\left[\frac{(2n)!}{(n!)^2}\right]^{1/n} = e^{\frac{1}{n}\log\frac{(2n)!}{(n!)^2}} = e^{\frac{1}{n}\left[\log(2n)! - 2\log n!\right]} \sim$$

$$\log(2n)! \sim 2n\log 2n \qquad \log n! \sim n\log n$$

$$\sim e^{\frac{1}{n}\left[2n\log 2n - 2n\log n\right]} = e^{2\log 2} = 4$$