BY GIMUSI 31/10/15

Limiti 12

Argomenti: Limiti di funzioni e successioni **Difficoltà:** ⋆⋆⋆

Prerequisiti: sviluppi di Taylor

Calcolare i limiti delle seguenti funzioni (tutti i limiti si intendono per $x \to 0$).

	a) Funzione	Limite	b) Funzione	Limite
1)	$\dfrac{x^2-2+2\cos x}{x^3}$	0	$\dfrac{x^2-2+2\cos x}{x^4}$	1/12
2)	$\dfrac{x^2-2+2\cos x}{x^5}$	N.E.	$\dfrac{x^2-2+2\cos x}{x^6}$	$+\infty$
3)	$\dfrac{x-\sin x}{x-\arctan x}$	1/2	$\dfrac{e^x+\cos x-2-x}{\arctan x-\sin x}$	-1
4)	$\dfrac{\sqrt[3]{1+x}+\sqrt[3]{1-x}-2}{1-\cos x}$	-5/8	$\dfrac{\sin x\cdot\sinh x\cdot\tan x}{x-\sin(x+x^3)}$	-6/5
5)	$\dfrac{e^x+\cos x-2-x}{\arctan x-\sin x}$	-2 = 3.b	$\dfrac{\log(1+\sin^2 x)-x^2}{\sin^2(\tanh^2 x)-x^5}$	-5/6
6)	$\dfrac{\sqrt[3]{1+x}+\sqrt[3]{1-x}-2}{1-\cos x+x^4}$	-5/8	$\dfrac{\sqrt[3]{1+\sin x}+\sqrt[3]{1-\sin x}-2}{1-\cos(\sinh x)+\sin^4 x}$	-5/8
7)	$\dfrac{(1+x+x^2)^{1/x}}{1-(3x+1)\cos\sqrt{x}}$	N.E.	$\dfrac{e^{x+x^3}-e^{\sinh x}}{2\log(1+x+x^2)-2x-3x^2}$	0
8)	$\dfrac{\cos(\sinh x)-\cos x}{x\tan(x^3+x)-\arctan x}$	0	$\dfrac{\sqrt{1+\sin(x^3)}-\sqrt{1+\sinh(x^3)}}{x^2\arctan(x\sin^6 x)+\lvert\sin^7(x^2)\rvert}$	-1/6

Calcolare i limiti delle seguenti successioni.

	a) Successione	Limite	b) Successione	Limite
9)	$n^3\left(\sin\dfrac{1}{n}-\sinh\dfrac{1}{n}\right)$	-1/3	$n^3\left(\sin\dfrac{1}{n^2}-\sin\dfrac{1}{n^2+n}\right)$	1
10)	$n^2\left(1-n\arctan\dfrac{1}{n}\right)$	1/3	$\dfrac{2\cos n+n^2-2}{n^4}$	0
11)	$n^n\sinh^n\dfrac{1}{n}$	1	$n^3\sin\dfrac{1}{n}-n^4\arctan\dfrac{1}{n^2}$	-1/6
12)	$\dfrac{\log(n^2+3)-2\log n}{\arctan(n+3)-\arctan n}$	1	$\left(\dfrac{n^2+3n+2}{n^2-7n+5}\right)^{(n^2+3)/(n-4)}$	e^{10}

1/6

Calcolare i limiti delle seguenti funzioni (tutti i limiti si intendono per $x \to 0$).

1.a) $$\frac{x^2-2+2\cos x}{x^3} = \frac{x^2-2+2-x^2+o(x^3)}{x^3} = \frac{o(x^3)}{x^3} \to 0$$

1.b) $$\frac{x^2-2+2\cos x}{x^4} = \frac{x^4/12+o(x^4)}{x^4} \to 1/12$$

2.a) $$\frac{x^2-2+2\cos x}{x^5} = \frac{x^4/12+o(x^5)}{x^5} \to N.E. \ (\pm\infty)$$

2.b) $$\frac{x^2-2+2\cos x}{x^6} = \frac{x^4/12-2x^6/6!+o(x^6)}{x^6} \to +\infty$$

3.a) $$\frac{x-\sin x}{x-\arctan x} = \frac{x^3/6+o(x^3)}{x^3/3+o(x^3)} \to 1/2$$

3.b) $$\frac{e^x+\cos x-2-x}{\arctan x-\sin x} = \frac{\cancel{1}+\cancel{x}+\cancel{x^2/2}+x^3/6+\cancel{1}-\cancel{x^2/2}-\cancel{2}-\cancel{x}+o(x^3)}{-x^3/3+x^3/6+o(x^3)} \to -1$$

4.a) $$\frac{\sqrt[3]{1+x}+\sqrt[3]{1-x}-2}{1-\cos x} = \frac{\cancel{1}+\cancel{x/3}-x^2/9+\cancel{1}-\cancel{x/3}-x^2/9\cancel{-2}+o(x^2)}{x^2/2+o(x^2)} \to -4/9$$

4.b) $$\frac{\sin x\cdot\sinh x\cdot\tan x}{x-\sin(x+x^3)} = \frac{(x-x^3/6+o(x^3))(x+x^3/6+o(x^3))(x-x^3/3+o(x^3))}{x-(x+x^3-(x+x^3)^3/6+o(x^3))} =$$

$$= \frac{x^3+o(x^5)}{x-(x+x^3-x^3/6+o(x^3))} = \frac{x^3+o(x^5)}{-5/6x^3+o(x^3)} \to -6/5$$

5.a) Vd. 3.b

5.b) $$\frac{\log(1+\sin^2 x)-x^2}{\sin^2(\tanh^2 x)-x^5} = \frac{\log(1+(x-x^3/6+o(x^3))^2)-x^2}{\sin^2((x-x^3/3+o(x^3))^2)-x^5} =$$

$$= \frac{x^2-x^4/3-x^4/2+o(x^4)-x^2}{\sin^2(x^2-2x^4/3+o(x^4))-x^5} = \frac{-5x^4/6+o(x^4)}{x^4+o(x^4)} \to -5/6$$

6.a) $$\frac{\sqrt[3]{1+x}+\sqrt[3]{1-x}-2}{1-\cos x+x^5} = \frac{\cancel{1}+\cancel{x/3}-x^2/9+\cancel{1}-\cancel{x/3}-x^2/9-\cancel{2}+o(x^2)}{x^2/2+o(x^2)} \to -5/9$$

6.b) $$\frac{\sqrt[3]{1+\sin x}+\sqrt[3]{1-\sin x}-2}{1-\cos(\sinh x)+\sin^5 x} = \frac{\sqrt[3]{1+x+o(x^2)}+\sqrt[3]{1-x+o(x^2)}-2}{1-\cos(x+o(x))+x^5+o(x^5)} =$$

$$= \frac{\cancel{1}+\cancel{x/3}-x^2/9+\cancel{1}-\cancel{x/3}-x^2/9+o(x^2)-\cancel{2}}{x^2/2+o(x^2)} = -5/9$$

7.a) $$\frac{(1+x+x^2)^{1/x}}{1-(3x+1)\cos\sqrt{x}} = \frac{e^{\frac{1}{x}\log(1+x+x^2)}}{1-(3x+1)(1-x/2+o(x))} =$$

$$= \frac{e^{\frac{1}{x}(x+x^2-(x+x^2)^2/2+o(x^2))}}{\cancel{1}-\cancel{1}+x/2-3x+o(x)} =$$

$$= \frac{e^{\frac{1}{x}(x+x^2-x^2/2+o(x^2))}}{-5x/2+o(x)} = \frac{e^{(1+x/2+o(x))}}{-5x/2+o(x)} =$$

$$= \frac{e\left(1+\frac{x}{2}+o(x)\right)}{-5x/2+o(x)} \to N.E.$$

7.b) $$\frac{e^{x+x^3}-e^{\sinh x}}{2\log(1+x+x^2)-2x-3x^2} =$$

$$= \frac{e^{x+x^3}-e^{x+x^3/6+o(x^3)}}{2(x+x^2-(x+x^2)^2/2+o(x^2))-2x-3x^2} =$$

$$= \frac{\cancel{1}+\cancel{x}+x^3+(\cancel{x+x^3})^2/2+\cancel{x^3/6}-\cancel{1}-\cancel{x}-x^3/6-(\cancel{x+x^3/6})^2/2-\cancel{x^3/6}+o(x^3)}{2(x+x^2-x^2/2+o(x^2))-2x-3x^2} =$$

$$= \frac{5x^3/6 + o(x^3)}{-2x^2 + o(x^2)} \to 0$$

8.a) $$\frac{\cos(\sinh x) - \cos x}{x \tan(x^3+x) - \arctan x} =$$

$$= \frac{\cos(x + x^3/6 + o(x^3)) - (1 - x^2/2 + x^4/24 + o(x^4))}{x\,(x + x^3 + x^3/3 + o(x^3)) - x - x^3/3 + o(x^3)} =$$

$$= \frac{\cancel{1} - (x + x^3/6)^2/2 + \cancel{x^4/24} \cancel{- 1} + x^2/2 - \cancel{x^4/24} + o(x^4)}{x^2 - x - x^3/3 + o(x^3)} =$$

$$= \frac{-x^4/6 + o(x^4)}{-x + x^2 - x^3/3 + o(x^3)} \to 0$$

8.b) $$\frac{\sqrt{1 + \sin(x^3)} - \sqrt{1 + \sinh(x^3)}}{x^2 \arctan(x \sin^6 x) + |\sin^7(x^2)|} =$$

$$= \frac{(1 + x^3 - x^9/6 + o(x^9))^{1/2} - (1 + x^3 + x^9/6 + o(x^9))}{x^2 \arctan(x^7 + o(x^7)) + |\,o(x^{14})\,|} =$$

$$= \frac{\cancel{1} + \cancel{x^3/2} - x^9/12 - \cancel{1} - \cancel{x^3/2} - x^9/12 + o(x^9)}{x^9 + o(x^9)} =$$

$$= \frac{-x^9/6 + o(x^9)}{x^9 + o(x^9)} \to -1/6$$

9.a) $$n^3\left(\sin\frac{1}{n}-\sinh\frac{1}{n}\right)=n^3\left(\cancel{\frac{1}{n}}-\frac{1}{6n^3}-\cancel{\frac{1}{n}}-\frac{1}{6n^3}+o\left(\frac{1}{n^3}\right)\right)=$$

$$=-\frac{1}{3}+o(1)\to -1/3$$

9.b) $$n^3\left(\sin\frac{1}{n^2}-\sin\frac{1}{n^2+n}\right)=n^3\left(\frac{1}{n^2}-\frac{1}{n^2+n}+o\left(\frac{1}{n^3}\right)\right)=$$

$$=n^3\frac{n}{n^2(n^2+n)}+o(1)\to 1$$

10.a) $$n^2\left(1-n\arctan\frac{1}{n}\right)=n^2\left(1-n\left(\frac{1}{n}-\frac{1}{3n^3}+o\left(\frac{1}{n^3}\right)\right)\right)=$$

$$=n^2\left(\cancel{1}-\cancel{1}+\frac{1}{3n^2}+o\left(\frac{1}{n^2}\right)\right)=\frac{1}{3}+o(1)\to 1/3$$

10.b) $$\frac{2\cos n+n^2-2}{n^5}=\underbrace{\frac{2\cos n}{n^5}}_{\to 0}+\underbrace{\frac{n^2-2}{n^5}}_{\to 0}\to 0$$

11.a) $$n^n\sin^n\frac{1}{n}=e^{n\log\left(n\sin\frac{1}{n}\right)}=$$

$$=e^{n\log\left(n\left(\frac{1}{n}-\frac{1}{6n^3}+o\left(\frac{1}{n^3}\right)\right)\right)}=$$

$$=e^{n\log\left(1-\frac{1}{6n^2}+o\left(\frac{1}{n^2}\right)\right)}=e^{n\left(-\frac{1}{6n^2}+o\left(\frac{1}{n^2}\right)\right)}=$$

$$=e^{-\frac{1}{6n}+o\left(\frac{1}{n}\right)}=1-\frac{1}{6n}+o\left(\frac{1}{n}\right)\to 1$$

11.b) $$n^3 \sin\left(\frac{1}{n}\right) - n^5 \arctan\left(\frac{1}{n^2}\right) = n^3\left(\frac{1}{n} - \frac{1}{6n^3} + o\left(\frac{1}{n^3}\right)\right) +$$

$$- n^5\left(\frac{1}{n^2} + o\left(\frac{1}{n^5}\right)\right) = \cancel{n}^2 - \frac{1}{6} - \cancel{n}^2 + o(1) = -1/6$$

12.a) $$\frac{\log(n^2+3) - 2\log n}{\arctan(n+3) - \arctan(n)} = \frac{\log(1+3/n^2)}{\arctan(1/n+3) - \arctan(1/n)} =$$

$$= \frac{3/n^2 + o(1/n^2)}{\frac{1}{n+3} - \frac{1}{n} + o(1/n^2)} = \frac{3/n^2 + o(1/n^2)}{\frac{-3}{n^2+3n} + o(1/n^2)} \to 1$$

12.b) $$\left(\frac{n^2+3n+2}{n^2-7n+5}\right)^{(n^2+3)/(n-5)} =$$

$$= e^{\frac{n^2+3}{n-5}\left[\log\left(1+\frac{3}{n}+\frac{2}{n^2}\right) - \log\left(1-\frac{7}{n}+\frac{5}{n^2}\right)\right]} =$$

$$= e^{\frac{n^2+3}{n-5}\left(\frac{3}{n}+\frac{7}{n}+o\left(\frac{1}{n}\right)\right)} = e^{\overset{\to 1}{\frac{n^2+3}{n^2-5n}}\left(\overset{\to 10}{10+o(1)}\right)} \to e^{10}$$