BY GIMUSI 10/01/15

Serie 2

Argomenti: convergenza di serie a termini di segno costante **Difficoltà:** ⋆⋆

Prerequisiti: criterio del rapporto, della radice e del confronto asintotico

Stabilire se le seguenti serie numeriche sono convergenti oppure no.

	a) Serie	Conv.?	b) Serie	Conv.?	c) Serie	Conv.?
1)	$\sum_{n=0}^{\infty} \frac{n+1}{2n-1}$	NO	$\sum_{n=0}^{\infty} \frac{n+1}{n^2-3}$	NO	$\sum_{n=0}^{\infty} \frac{n^2+2}{n^4-4}$	SÌ
2)	$\sum_{n=0}^{\infty} \frac{n}{n^3+1}$	SÌ	$\sum_{n=0}^{\infty} \frac{n}{\sqrt{n^3+4}}$	NO	$\sum_{n=0}^{\infty} \frac{\sqrt[3]{2+n^2}}{\sqrt[4]{7+n^7}}$	SÌ
3)	$\sum_{n=1}^{\infty} \arctan \frac{1}{n}$	NO	$\sum_{n=1}^{\infty} \sin \frac{1}{\sqrt{n}}$	NO	$\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{1}{n}$	SÌ
4)	$\sum_{n=1}^{\infty} n \sin \frac{1}{n^2}$	NO	$\sum_{n=1}^{\infty} \sqrt{n} \sin \frac{1}{n^2}$	NO	$\sum_{n=1}^{\infty} \left(1-\cos \frac{1}{n}\right)$	SÌ
5)	$\sum_{n=1}^{\infty} \left(\sqrt[n]{7}-1\right)$	NO	$\sum_{n=1}^{\infty} \left(\sqrt[n]{7}-1\right)^2$	SÌ	$\sum_{n=1}^{\infty} \sin \frac{n+3}{n^4+1}$	SÌ

Stabilire per quali valori del parametro reale $\alpha > 0$ le seguenti serie numeriche convergono.

	a) Serie	α	b) Serie	α	c) Serie	α
6)	$\sum_{n=0}^{\infty} \frac{n+1}{n^\alpha+3}$	>2	$\sum_{n=0}^{\infty} \frac{n^\alpha+5}{n^8+7}$	<7	$\sum_{n=0}^{\infty} \frac{n^\alpha+5}{n^{3\alpha}+7}$	$>1/2$
7)	$\sum_{n=1}^{\infty} \frac{n^2+1}{n^4+n^\alpha}$	>0	$\sum_{n=0}^{\infty} \frac{n+1}{\sqrt{n^\alpha+2}}$	>5	$\sum_{n=0}^{\infty} \frac{n^\alpha+1}{\sqrt[3]{n^7+1}}$	$<5/3$
8)	$\sum_{n=1}^{\infty} \sin \frac{1}{n^\alpha}$	>1	$\sum_{n=1}^{\infty} \left(\sin \frac{1}{n}\right)^\alpha$	>1	$\sum_{n=1}^{\infty} \sin^2 \left(\frac{n^{3\alpha}}{n+2}\right)$	—
9)	$\sum_{n=1}^{\infty} \frac{\alpha^n}{n^4}$	≤ 1	$\sum_{n=1}^{\infty} \frac{2^n}{n^\alpha}$	—	$\sum_{n=1}^{\infty} \sqrt{n} \sin \frac{1}{n^\alpha}$	$>3/2$
10)	$\sum_{n=1}^{\infty} \frac{1}{2^n+n^\alpha}$	>0	$\sum_{n=1}^{\infty} \frac{1}{n+\alpha^n}$	>1	$\sum_{n=1}^{\infty} \frac{1}{n^2+\alpha^n}$	>0

1/6

1.a) $\sum_{n=0}^{\infty} \frac{n+1}{2n-1}$ NON CONVERGE $\qquad a_n = \frac{n+1}{2n-1} \to 1/2$

1.b) $\sum_{n=0}^{\infty} \frac{n+1}{n^2-3}$ NON CONVERGE PER CONF. ASINTOTICO

$$a_n = \frac{n+1}{n^2-3} \qquad b_n = \frac{1}{n} \qquad \frac{a_n}{b_n} = \frac{n^2+n}{n^2-3} \to 1$$

1.c) $\sum_{n=0}^{\infty} \frac{n^2+2}{n^4-4}$ CONVERGE PER CONF. ASINTOTICO

$$a_n = \frac{n^2+2}{n^4-4} \qquad b_n = \frac{1}{n^2} \qquad \frac{a_n}{b_n} = \frac{n^4+2n^2}{n^4-4} \to 1$$

2.a) $\sum_{n=0}^{\infty} \frac{n}{n^3+1}$ CONVERGE PER CONF. ASINTOTICO

$$a_n = \frac{n}{n^3+1} \qquad b_n = \frac{1}{n^2} \qquad \frac{a_n}{b_n} = \frac{n^3}{n^3+1} \to 1$$

2.b) $\sum_{n=0}^{\infty} \frac{n}{\sqrt{n^3+4}}$ NON CONVERGE PER CONF. ASINTOTICO

$$a_n = \frac{n}{\sqrt{n^3+4}} \qquad b_n = \frac{1}{\sqrt{n}} \qquad \frac{a_n}{b_n} = \frac{\sqrt{n^3}}{\sqrt{n^3+4}} \to 1$$

2.c) $\sum_{n=0}^{\infty} \frac{\sqrt[3]{2+n^2}}{\sqrt[3]{7+n^7}}$ CONVERGE PER CONF. ASINTOTICO

$$a_n = \frac{\sqrt[3]{2+n^2}}{\sqrt[3]{7+n^7}} \qquad b_n = \frac{1}{\sqrt[3]{n^5}} \qquad \frac{a_n}{b_n} = \frac{\sqrt[3]{2n^5+n^7}}{\sqrt[3]{7+n^7}} \to 1$$

3.a) $\sum_{n=1}^{\infty} \mathrm{ARCTAN}\, \frac{1}{n}$ NON CONVERGE PER CONF. ASINTOTICO

$$a_n = \mathrm{ARCTAN}\, \frac{1}{n} \qquad b_n = \frac{1}{n} \qquad \frac{a_n}{b_n} = \frac{\mathrm{ARCTAN}\, \frac{1}{n}}{1/n} \to 1$$

3.b) $\sum_{n=1}^{\infty} \mathrm{SIN}\, \frac{1}{\sqrt{n}}$ NON CONVERGE PER CONF. ASINTOTICO

$$a_n = \mathrm{SIN}\, \frac{1}{\sqrt{n}} \qquad b_n = \frac{1}{\sqrt{n}} \qquad \frac{a_n}{b_n} = \frac{\mathrm{SIN}\, 1/\sqrt{n}}{1/\sqrt{n}} \to 1$$

3.c) $\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{1}{n}$ CONVERGE PER CONF. ASINTOTICO

$$a_n = \frac{1}{n} \sin \frac{1}{n} \qquad b_n = \frac{1}{n^2} \qquad \frac{a_n}{b_n} = \frac{\sin 1/n}{1/n} \to 1$$

4.a) $\sum_{n=1}^{\infty} n \sin \frac{1}{n^2}$ NON CONVERGE PER CONF. ASINTOTICO

$$a_n = n \sin \frac{1}{n^2} \qquad b_n = \frac{1}{n} \qquad \frac{a_n}{b_n} = \frac{\sin 1/n^2}{1/n^2} \to 1$$

4.b) $\sum_{n=1}^{\infty} \sqrt{n} \sin \frac{1}{n^2}$ CONVERGE PER CONF. ASINTOTICO

$$a_n = \sqrt{n} \sin \frac{1}{n^2} \qquad b_n = \frac{1}{\sqrt{n^3}} \qquad \frac{a_n}{b_n} = \frac{\sin 1/n^2}{1/n^2} \to 1$$

4.c) $\sum_{n=1}^{\infty} \left(1 - \cos \frac{1}{n}\right)$ CONVERGE PER CONF. ASINTOTICO

$$a_n = 1 - \cos \frac{1}{n} \qquad b_n = \frac{1}{n^2} \qquad \frac{a_n}{b_n} = \frac{1 - \cos(1/n)}{1/n^2} \to 1/2$$

5.a) $\sum_{n=1}^{\infty} \left(\sqrt[n]{7} - 1\right)$ NON CONVERGE PER CONF. ASINTOTICO

$$a_n = \sqrt[n]{7} - 1 \qquad b_n = \frac{1}{n} \qquad \frac{a_n}{b_n} = \frac{7^{1/n} - 1}{1/n} \to \log 7$$

5.b) $\sum_{n=1}^{\infty} \left(\sqrt[n]{7} - 1\right)^2$ CONVERGE PER CONF. ASINTOTICO

$$a_n = \left(\sqrt[n]{7} - 1\right)^2 \qquad b_n = \frac{1}{n^2} \qquad \frac{a_n}{b_n} = \left(\frac{7^{1/n} - 1}{1/n}\right)^2 \to \log^2 7$$

5.c) $\sum_{n=1}^{\infty} \sin \frac{n+3}{n^5+1}$ CONVERGE PER CONF. ASINTOTICO

$$a_n = \sin \frac{n+3}{n^5+1} \qquad b_n = \frac{n+3}{n^5+1} \qquad \frac{a_n}{b_n} = \frac{\sin \frac{n+3}{n^5+1}}{\frac{n+3}{n^5+1}} \to 1$$

$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{n+3}{n^5+1}$ CONVERGE PER CONF. ASINTOTICO

$$b_n = \frac{n+3}{n^5+1} \qquad c_n = \frac{1}{n^3} \qquad \frac{b_n}{c_n} = \frac{n^5 + n^3}{n^5+1} \to 1$$

6.a) $\sum_{n=1}^{\infty} \frac{n+1}{n^\alpha+3}$ PER $\alpha>2$ CONVERGE PER CONF. ASINTOTICO

$a_n = \frac{n+1}{n^\alpha+3}$ (C.N. $\alpha>1$) $\quad b_n = \frac{1}{n^{\alpha-1}} \quad \frac{a_n}{b_n} = \frac{n^\alpha+n^{\alpha-1}}{n^\alpha+3} \to 1$

6.b) $\sum_{n=1}^{\infty} \frac{n^\alpha+5}{n^8+7}$ PER $\alpha<7$ CONVERGE PER CONF. ASINTOTICO

$a_n = \frac{n^\alpha+5}{n^8+7}$ (C.N. $\alpha<8$) $\quad b_n = \frac{1}{n^{8-\alpha}} \quad \frac{a_n}{b_n} = \frac{n^8+5n^{8-\alpha}}{n^8+7} \to 1$

6.c) $\sum_{n=1}^{\infty} \frac{n^\alpha+5}{n^{3\alpha}+7}$ PER $\alpha>1/2$ CONVERGE PER CONF. ASINTOTICO

$a_n = \frac{n^\alpha+5}{n^{3\alpha}+7}$ ($\to 0\ \forall\alpha$) $\quad b_n = \frac{1}{n^{2\alpha}} \quad \frac{a_n}{b_n} = \frac{n^{3\alpha}+5n^{2\alpha}}{n^{3\alpha}+7} \to 1$

7.a) $\sum_{n=1}^{\infty} \frac{n^2+1}{n^4+n^\alpha}$ PER $\alpha>0$ CONVERGE PER CONF. ASINTOTICO

$a_n = \frac{n^2+1}{n^4+n^\alpha}$ ($\to 0\ \forall\alpha$) $\quad b_n = \frac{1}{n^2} \quad \frac{a_n}{b_n} = \frac{n^4+n^2}{n^4+n^\alpha} \to \begin{cases} 1 & \alpha<4 \\ 1/2 & \alpha=4 \\ 0 & \alpha>4 \end{cases}$

$\alpha>4 \quad \frac{a_n}{b_n} \to 0 \implies a_n \le b_n$ DEF.

7.b) $\sum_{n=0}^{\infty} \frac{n+1}{\sqrt{n^\alpha+2}}$ PER $\alpha>4$ CONVERGE PER CONF. ASINTOTICO

$a_n = \frac{n+1}{\sqrt{n^\alpha+2}}$ (C.N. $\alpha>1$) $\quad b_n = \frac{1}{n^{\alpha/2-1}} \quad \frac{a_n}{b_n} = \frac{n^{\alpha/2}+n^{\alpha/2-1}}{(n+2)^{\alpha/2}} \to 1$

7.c) $\sum_{n=0}^{\infty} \frac{n^\alpha+1}{\sqrt[3]{n^7+1}}$ PER $\alpha<4/3$ CONVERGE PER CONF. ASINTOTICO

$a_n = \frac{n^\alpha+1}{\sqrt[3]{n^7+1}}$ (C.N. $\alpha<7/3$) $\quad b_n = \frac{1}{n^{7/3-\alpha}} \quad \frac{a_n}{b_n} = \frac{n^{7/3}+n^{7/3-\alpha}}{(n^7+1)^{1/3}} \to 1$

8.a) $\sum_{n=1}^{\infty} \sin\frac{1}{n^\alpha}$ PER $\alpha>1$ CONVERGE PER CONF. ASINTOTICO

$a_n = \sin\frac{1}{n^\alpha}$ ($\to 0\ \forall\alpha$) $\quad b_n = \frac{1}{n^\alpha} \quad \frac{a_n}{b_n} = \frac{\sin(1/n^\alpha)}{1/n^\alpha} \to 1$

8.b) $\sum_{n=1}^{\infty} \left(\sin\frac{1}{n}\right)^\alpha$ PER $\alpha>1$ CONVERGE PER CONF. ASINTOTICO

$a_n = \left(\sin\frac{1}{n}\right)^\alpha$ ($\to 0\ \forall\alpha$) $\quad b_n = \frac{1}{n^\alpha} \quad \frac{a_n}{b_n} = \left(\frac{\sin(1/n)}{1/n}\right)^\alpha \to 1$

8.c) $\sum_{n=1}^{\infty} \sin^2\left(\frac{n^{3\alpha}}{n+2}\right)$ NON CONVERGE

$a_n = \sin^2\left(\frac{n^{3\alpha}}{n+2}\right)$ (C.N. $\alpha < 1/3$) $b_n = \left(\frac{n^{3\alpha}}{n+2}\right)^2$ $\frac{a_n}{b_n} \to 1$ $\alpha < 1/3$

$\sum b_n$ NON CONV. $\forall \alpha > 0$

$c_n = \frac{1}{\sqrt{n}}$ $\frac{b_n}{c_n} = \left(n^{3\alpha}\frac{n}{n+2}\right)^2 \to +\infty$ $\forall \alpha > 0$

9.a) $\sum_{n=1}^{\infty} \frac{\alpha^n}{n^5}$ PER $\alpha \le 1$ CONVERGE PER CONF. ASINTOTICO

$a_n = \frac{\alpha^n}{n^5}$ (C.N. $\alpha \le 1$) $b_n = \frac{1}{n^5}$ $\frac{a_n}{b_n} \to \begin{cases} 0 & \alpha < 1 \\ 1 & \alpha = 1 \end{cases}$

9.b) $\sum_{n=1}^{\infty} \frac{2^n}{n^\alpha}$ NON CONVERGE

$a_n = \frac{2^n}{n^\alpha}$ $\sqrt[n]{a_n} = \frac{2}{\sqrt[n]{n^\alpha}} \to 2$ $\forall \alpha > 0$

9.c) $\sum_{n=1}^{\infty} \sqrt{n}\,\sin\frac{1}{n^\alpha}$ PER $\alpha > 3/2$ CONVERGE PER CONF. ASINTOTICO

$a_n = \sqrt{n}\,\sin\frac{1}{n^\alpha}$ $b_n = \frac{\sqrt{n}}{n^\alpha}$ $\frac{a_n}{b_n} = \frac{\sin(1/n^\alpha)}{1/n^\alpha} \to 1$ $\forall \alpha > 0$

$\sum b_n$ CONV. PER $\alpha - \frac{1}{2} > 1$ $\alpha > 3/2$

10.a) $\sum_{n=1}^{\infty} \frac{1}{2^n + n^\alpha}$ CONVERGE $\forall \alpha > 0$

$a_n = \frac{1}{2^n + n^\alpha}$ $b_n = \frac{1}{2^n}$ $\frac{a_n}{b_n} = \frac{2^n}{2^n + n^\alpha} \to 1$ $\forall \alpha$

$\sum b_n = 1$ (SERIE GEOMETRICA)

10.b) $\sum_{n=1}^{\infty} \frac{1}{n + \alpha^n}$ CONVERGE $\alpha > 1$

$\alpha \le 1$ $a_n = \frac{1}{n + \alpha^n}$ $b_n = \frac{1}{n}$ $\frac{a_n}{b_n} \to 1$ $\forall \alpha \le 1$

$\alpha > 1$ $b_n = \frac{1}{\alpha^n}$ $\frac{a_n}{b_n} \to 1$ $\sum b_n = \frac{\alpha}{1-\alpha}$

10.c) $\sum_{n=1}^{\infty} \frac{1}{n^2+\alpha^n}$ CONVERGE $\forall \alpha > 0$

$\alpha \leq 1$ $\quad a_n = \frac{1}{n^2+\alpha^n} \quad b_n = \frac{1}{n^2} \quad \frac{a_n}{b_n} \to 1 \quad \forall \alpha \leq 1$

$\alpha > 1$ $\quad b_n = \frac{1}{\alpha^n} \quad \frac{a_n}{b_n} \to 1 \quad \sum b_n = \frac{\alpha}{1-\alpha}$