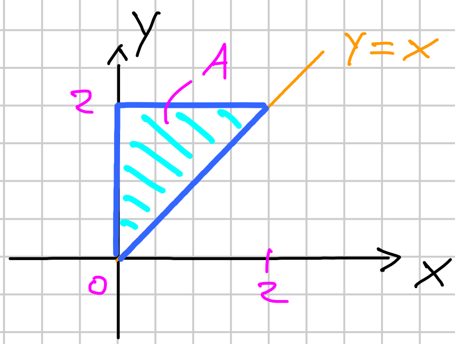
BY GIMUSI 07/06/15

Solido di rotazione:

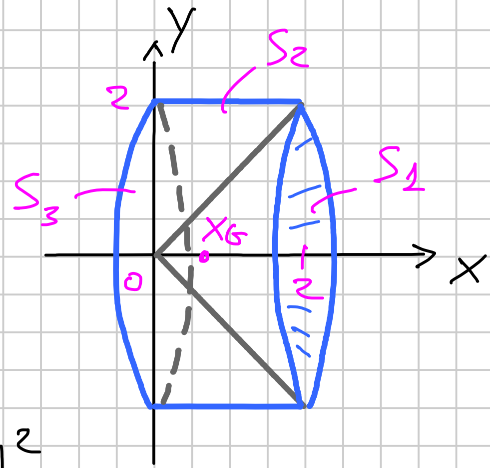


$0 \le y \le 2 \qquad 0 \le x \le y$

$A = \frac{1}{2} 2 \cdot 2 = 2$

$y_G = 2 - \frac{2}{3} = \frac{4}{3}$ ≡ Y BARICENTRO A

Rotazione attorno asse X



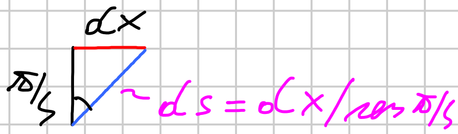
$V = 2\pi y_G A = 2\pi \cdot \frac{4}{3} \cdot 2 = \frac{16}{3}\pi$

$$x_G = \frac{1}{V}\int_0^2 x \cdot \pi (2^2 - x^2)\, dx =$$

$$= \frac{\pi}{V}\int_0^2 4x - x^3\, dx = \frac{3}{16}\left[2x^2 - \frac{x^4}{4}\right]_0^2 =$$

$$= \frac{3}{16}(8-4) = \frac{12}{16} = \frac{3}{4} \quad \rightsquigarrow \quad G_V = \left(\frac{3}{4}, 0, 0\right)$$

$S = S_1 + S_2 + S_3$



$$\begin{cases} S_1 = 2\pi \cdot 1 \cdot 2\sqrt{2} = 4\pi\sqrt{2} \quad \text{(GULDINO)} \\ S_1 = \int_0^2 2\pi x\, ds = 2\pi \int_0^2 \frac{x}{\cos \pi/4}\, dx = 2\sqrt{2}\pi \left[\frac{x^2}{2}\right]_0^2 = 4\pi\sqrt{2} \end{cases}$$

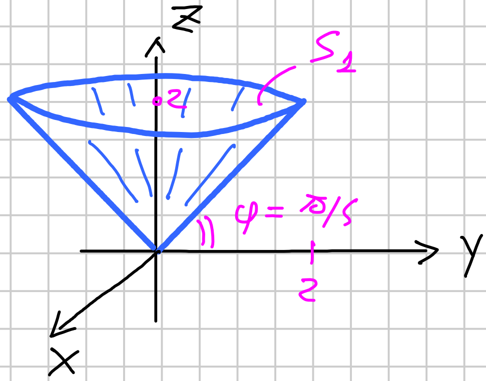
$$\begin{cases} S_2 = 2\pi \cdot 2 \cdot 2 = 8\pi \quad \text{(GULDINO)} \\ S_2 = \int_0^2 2\pi \cdot 2\, dx = 8\pi \end{cases}$$

$S_3 = \pi \cdot 2^2 = 4\pi$

$\rightsquigarrow S = 4\pi\sqrt{2} + 8\pi + 4\pi = 4\pi\sqrt{2} + 12\pi$

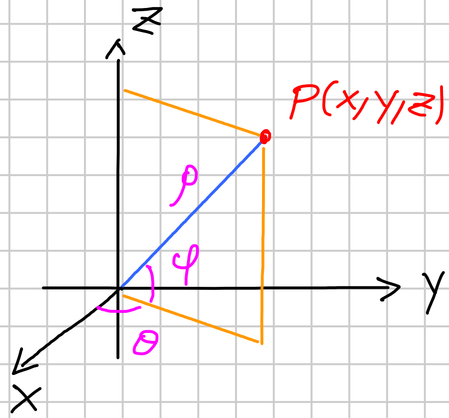
CALCOLO DI S_1 IN COORDINATE SFERICHE

MODO 1 - METODO GENERALE



$$S_1 = \{(x,y,z) = \left(\rho \tfrac{\sqrt{2}}{2}\cos\theta,\ \rho\tfrac{\sqrt{2}}{2}\sin\theta,\ \rho\tfrac{\sqrt{2}}{2}\right),\quad 0 \le \rho \le 2\sqrt{2},\ 0 \le \theta \le 2\pi\}$$

$$A = \int_{S_1} d\sigma = \int_{S_1} \sqrt{M_1^2 + M_2^2 + M_3^2}\, d\rho\, d\theta$$



$$\Phi(\rho,\theta) = \left(\rho\tfrac{\sqrt{2}}{2}\cos\theta,\ \rho\tfrac{\sqrt{2}}{2}\sin\theta,\ \rho\tfrac{\sqrt{2}}{2}\right)$$

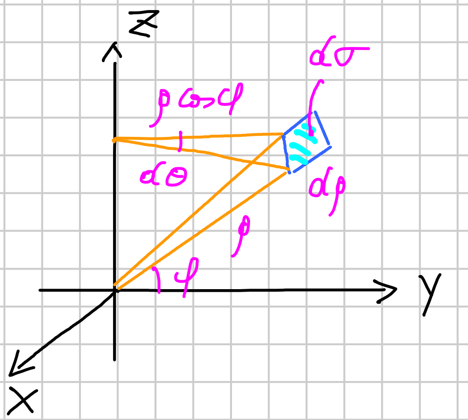
$$\begin{cases} \Phi_\rho(\rho,\theta) = \left(\tfrac{\sqrt{2}}{2}\cos\theta,\ \tfrac{\sqrt{2}}{2}\sin\theta,\ \tfrac{\sqrt{2}}{2}\right) \\ \Phi_\theta(\rho,\theta) = \left(-\rho\tfrac{\sqrt{2}}{2}\sin\theta,\ \rho\tfrac{\sqrt{2}}{2}\cos\theta,\ 0\right) \end{cases}$$

$$\leadsto M_1 = -\tfrac{1}{2}\rho\cos\theta,\quad M_2 = \tfrac{1}{2}\rho\sin\theta,\quad M_3 = \tfrac{1}{2}\rho(\cos^2\theta + \sin^2\theta) = \tfrac{1}{2}\rho$$

$$\sqrt{M_1^2 + M_2^2 + M_3^2} = \left(\tfrac{1}{4}\rho^2\cos^2\theta + \tfrac{1}{4}\rho^2\sin^2\theta + \tfrac{1}{4}\rho^2\right)^{1/2} = \tfrac{\sqrt{2}}{2}\rho$$

$$A = \int_0^{2\pi}\int_0^{2\sqrt{2}} \tfrac{\sqrt{2}}{2}\rho\, d\rho\, d\theta = \cancel{2}\pi\tfrac{\sqrt{2}}{\cancel{2}}\left[\tfrac{1}{2}\rho^2\right]_0^{2\sqrt{2}} = \pi\sqrt{2}\,(4-0) = 4\pi\sqrt{2}$$

MODO 2 - METODO DIRETTO



$$\int_{S_1} d\sigma = \int_0^{2\pi}\int_0^{2\sqrt{2}} \rho\tfrac{\sqrt{2}}{2}\, d\rho\, d\theta = 4\pi\sqrt{2}$$

$d\sigma = \rho\cos\varphi\, d\rho\, d\theta$