Integrali 2

Argomenti: Integrali in una variabile **Difficoltà**: $\star\star$

Prerequisiti: Primitive elementari, significato geometrico, spezzamento del dominio

Calcolare l'integrale delle seguenti funzioni sugli insiemi indicati. Indicare "N.P." se l'integranda non è limitata nell'insieme proposto (in questo caso l'esercizio diventa un esercizio sugli integrali impropri) e "N.S." se la richiesta non ha senso.

Funzione	**Ins.**	**Integrale**	**Ins.**	**Integrale**	**Ins.**	**Integrale**
$\lvert x\rvert$	$[0,1]$	$1/2$	$[-1,0]$	$1/2$	$[-1,2]$	$5/2$
$\lvert 2-x\rvert$	$[0,2]$	2	$[0,3]$	$5/2$	$[1,3]$	1
$x+\lvert x\rvert$	$[0,1]$	1	$[-1,0]$	0	$[-1,2]$	4
$\sqrt{\lvert x\rvert}$	$[0,1]$	$2/3$	$[-1,1]$	$4/3$	$[-1,0]$	$2/3$
$\sqrt{\lvert x-2\rvert}$	$[0,2]$	$4\sqrt{2}/3$	$[0,3]$	$4\sqrt{2}/3+2/3$	$[-2,2]$	$16/3$
$\lvert x^2-4\rvert$	$[0,2]$	$16/3$	$[0,3]$	$23/3$	$[-1,4]$	$59/3$
$\lvert\cos(2x)\rvert$	$[0,\pi]$	2	$[0,\pi/2]$	1	$[\pi/6,\pi/2]$	$1-\sqrt{3}/4$
$\dfrac{1}{\lvert x\rvert}$	$[-1,1]$	NP $(+\infty)$	$[-e,-1]$	1	$[-e^2,-e]$	1
$\lvert x^3\rvert-3$	$[-1,0]$	$-11/4$	$[-1,1]$	$-11/2$	$[-1,2]$	$-19/4$
$\lvert\sin x\rvert$	$[-\pi,\pi]$	4	$[-\pi,-\pi/2]$	1	$[-3\pi/4,0]$	$\sqrt{2}/2+1$
$\sin(\lvert x\rvert)$	$[-\pi,\pi]$	4	$[-2\pi,0]$	0	$[0,\pi]$	2
$\sin x\cos x$	$[0,\pi/2]$	$1/2$	$[\pi/2,\pi]$	$-1/2$	$[-\pi/2,0]$	$-1/2$
$\lvert\sin x\cos x\rvert$	$[0,\pi/2]$	$1/2$	$[\pi/2,\pi]$	$1/2$	$[-\pi/2,\pi]$	$3/2$
$\sqrt{1-x^2}$	$[-1,1]$	$\pi/2$	$[-1,0]$	$\pi/4$	$[0,2]$	N.S
$\cos(2x)\cos(7x)$	$[0,\pi/2]$	$7/45$	$[0,\pi/4]$	$-\sqrt{2}/45$	$[\pi/6,\pi/3]$	$1/180-\sqrt{3}/20$
$\cos(2x)\sin(7x)$	$[0,\pi/2]$	$7/45$	$[0,\pi/4]$	$7/45+\sqrt{2}/45$	$[\pi/6,\pi/3]$	$1/180-\sqrt{3}/20$

INTEGRALI 2 Test n. 37 Esercizi di A.M. (Parte A)

1) $|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$ $\quad \int x\,dx = \frac{x^2}{2}$; $\int -x\,dx = -\frac{x^2}{2}$

si risolvono geometricamente con i "quadretti"

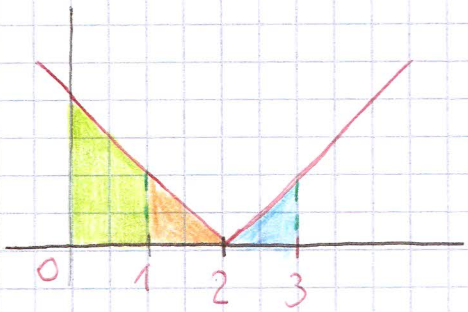
$[0, 1] = 1/2$

$[-1, 0] = 1/2$

$[-1\ 2] = 5/2$

$$\int_{-1}^{0} -x\,dx = -\frac{x^2}{2}\Big]_{-1}^{0} = F(0) - F(-1) = 0 - \left(-\frac{1}{2}\right) = 1/2$$

2) $|2-x| = \begin{cases} 2-x & \text{se } 2-x \geq 0 \quad x \leq 2 \\ -2+x & \text{se } 2-x < 0 \quad x > 2 \end{cases}$



$[0, 2] = 2$

$[0, 3] = 5/2$

$[1, 3] = 1$

$$\int_{1}^{3} |2-x|\,dx = \int_{1}^{2} (2-x)\,dx + \int_{2}^{3} (-2+x)\,dx = 2x - \frac{x^2}{2}\Big]_{1}^{2} + \left[-2x + \frac{x^2}{2}\right]_{2}^{3}$$

5

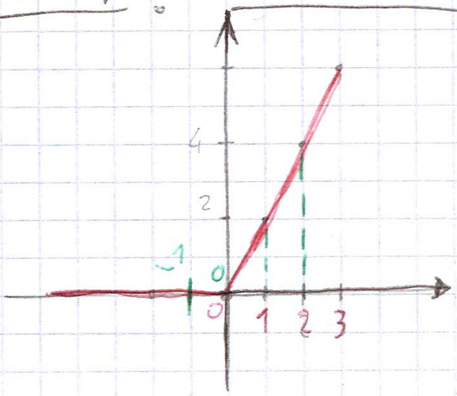
$$= F(2) - F(1) + F(3) - F(2) = 2 - \left(2 - \frac{1}{2}\right) + \left(-6 + 9/2\right) - (-2) = 5 - 6 + 2 = 1$$

geometricamente è molto più semplice

3) $x + |x|$

$= x + x = 2x \quad x \geq 0$

$= 0 \quad x < 0$



per $x \geq 0$

$$\int 2x\,dx = x^2 \;;\quad [0,1] \quad \left[x^2\right]_0^1 = F(1)-F(0) = 1$$

$[-1,0] = 0$; $[-1,2]$ da $[0,-1] = 0$

da $[0,2] = \left[x^2\right]_0^2 = F(2)-F(0) = 4$

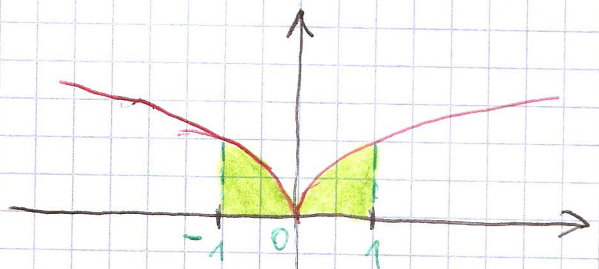
4) $\sqrt{|x|}$ — $\sqrt{x}$ se $x \geq 0$; $\sqrt{-x}$ se $x \leq 0$

$$\int \sqrt{x}\,dx = \frac{2x^{3/2}}{3}$$

$[0,1]$ $\left[\frac{2x^{3/2}}{3}\right]_0^1 = F(1)-F(0) = \frac{2}{3} - 0 = \frac{2}{3}$

$[-1,1]$ $= \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$

$[-1,0]$ $= \frac{2}{3}$

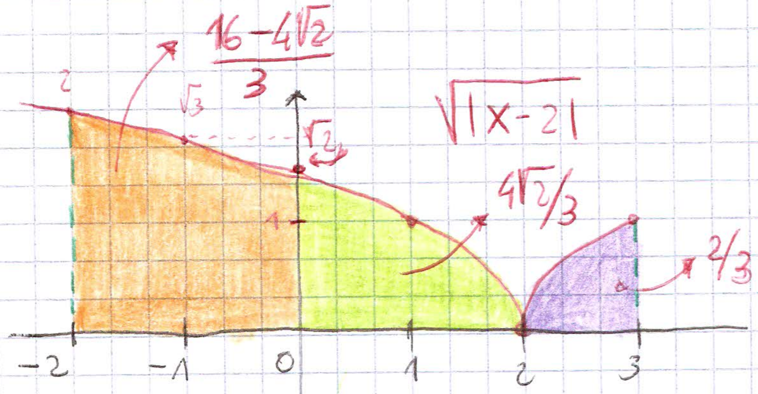


geometricamente calcolato il primo la soluzione nei 2 insiemi $[-1,1]$ e $[-1,0]$ è evidente.
i conteggi vanno fatti spezzando l'integrale con segno $\int$.

5) $\sqrt{|x-2|}$ — $\sqrt{x-2}$ se $x-2 \geq 0$; $x \geq 2$ — $\sqrt{-x+2}$ se $x-2<0$; $x<2$

$$\int \sqrt{x-2}\,dx = \frac{2}{3}\sqrt{(x-2)^3} \;, \quad \int \sqrt{-x+2}\,dx = -\frac{2}{3}\left(\sqrt{(-x+2)}\right)^3$$

per $x \geq 2$ — per $x < 2$



$$\left[-\frac{2}{3}\sqrt{(-x+2)^3}\right]_0^2 = F(2)-F(0) = -\left(-\frac{2}{3}\sqrt{2^3}\right) = 4\sqrt{2}/3$$

$$\int_0^3 \sqrt{|x-2|}\,dx = \int_0^2 \sqrt{|x-2|}\,dx + \int_2^3 \sqrt{|x-2|}\,dx =$$

$$= 4\sqrt{2}/3 + \left[\frac{2}{3}\sqrt{(x-2)^3}\right]_2^3 = F(3)-F(2) = \frac{2}{3} = 4\sqrt{2}/3 + 2/3$$

$$\left[-\frac{2}{3}\sqrt{(-x+2)^3}\right]_{-2}^2 = F(2)-F(-2) = 0-\left(-\frac{2}{3}\sqrt{4^3}\right) = \frac{2}{3}\cdot 8 = 16/3$$

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6) $|x^2-4|$

x^2-4 se $x^2-4 \ge 0$ $\quad x^2 \ge 4$ $\quad [+2, +\infty) \cup (-\infty, -2]$

x^2-4 se $x^2-4 \le 0$ $\quad x^2 < 4$ $\quad (-2, +2)$

$$\int_0^2 |x^2-4|\,dx = \int_0^2 (-x^2+4)\,dx = -\frac{1}{3}x^3 + 4x$$

$$\left[-\frac{1}{3}x^3+4x\right]_0^2 = F(2)-F(0) = -\frac{1}{3}\cdot 8 + 8 = \boxed{\frac{16}{3}}$$

$$\int_0^3 |x^2-4|\,dx = \int_0^2 |x^2-4|\,dx + \int_2^3 |x^2-4|\,dx =$$

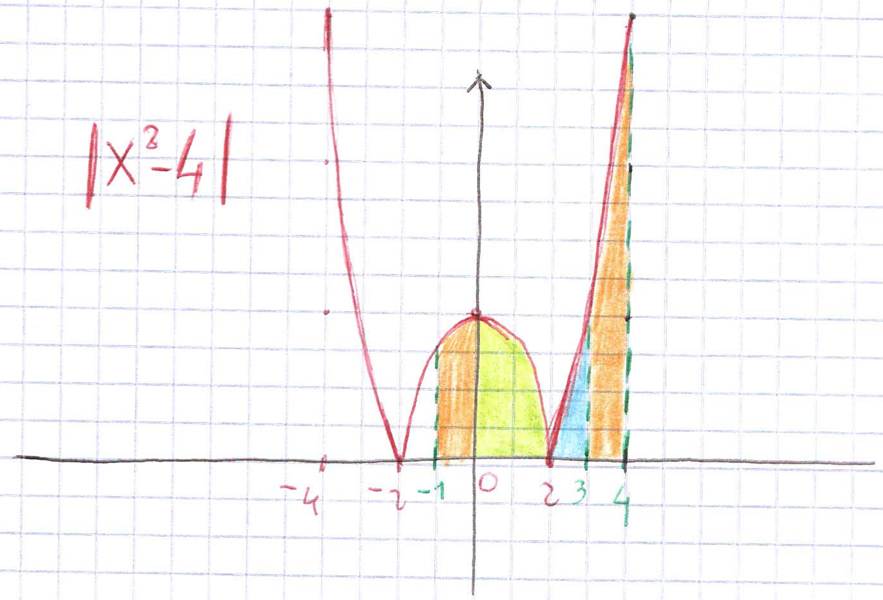
$$= \frac{16}{3} + \left[\frac{1}{3}x^3 - 4x\right]_2^3 = \frac{16}{3} + \left(-3 + \frac{16}{3}\right) = \frac{16}{3} + \frac{7}{3} = \boxed{23/3}$$

$$\int_{-1}^4 |x^2-4|\,dx = \int_{-1}^2 |x^2-4|\,dx + \int_2^4 |x^2-4|\,dx$$

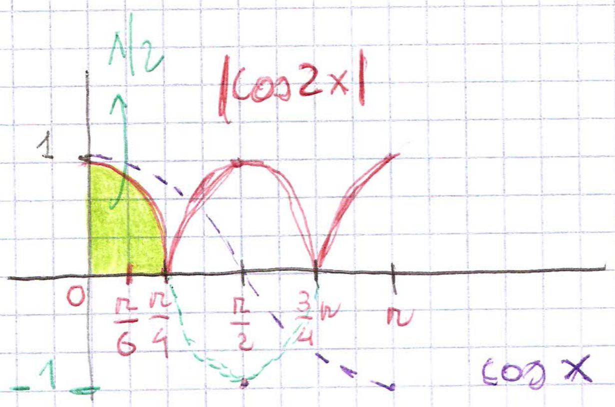
$$\left[-\frac{1}{3}x^3 + 4x\right]_{-1}^2 + \left[\frac{1}{3}x^3 - 4x\right]_2^4 = \left[F(2) - F(1)\right] + \left[F(4) - F(2)\right]$$

$$\left(-\frac{8}{3} + 8\right) - \left(\frac{1}{3} - 4\right) + \left(\frac{64}{3} - 16\right) - \left(\frac{8}{3} - 8\right) =$$

$$= \frac{16}{3} + \frac{11}{3} + \frac{16}{3} + \frac{16}{3} = 59/3$$



7) $|\cos(2x)|$



$$\int_0^{\pi} |\cos 2x|\, dx \quad \text{essendo} \quad |\cos 2x| \begin{cases} \cos 2x & \text{per } x \in \left[0, \frac{\pi}{4}\right] \cup \left[\frac{3}{4}\pi, \pi\right] \\ -\cos 2x & \text{per } x \in \left[\frac{\pi}{4}, \frac{3}{4}\pi\right] \end{cases}$$

$$\int \cos 2x\, dx = \frac{1}{2} \sin 2x$$

$$\int_0^{\pi} |\cos 2x|\, dx = \int_0^{\pi/4} \cos 2x\, dx + \int_{\pi/4}^{\frac{3}{4}\pi} -\cos 2x\, dx + \int_{3/4\pi}^{\pi} \cos 2x\, dx =$$

geometricamente calcolato il primo integrale tra 0 e $\pi/4$ si ha che l'integrale tra 0 e π è il primo valore moltiplicato per 4 cioè $1/2 \cdot 4 = 2$

$$= \left[\frac{1}{2} \sin 2x\right]_0^{\pi/4} + \left[-\frac{1}{2} \sin 2x\right]_{\pi/4}^{\frac{3}{4}\pi} + \left[\frac{1}{2} \sin 2x\right]_{\frac{3}{4}\pi}^{\pi} =$$

$$= \frac{1}{2} \cdot 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$$

$\left[0, \pi/2\right]$ in questo intervallo del grafico si ha $= 1$

$$\int_{\pi/6}^{\pi/2} |\cos 2x|\, dx = \int_{\pi/6}^{\pi/4} \cos 2x\, dx + \int_{\pi/4}^{\pi/2} - \cos 2x\, dx$$

$$\left[\frac{1}{2}\sin 2x\right]_{\pi/6}^{\pi/4} - \left[\frac{1}{2}\sin 2x\right]_{\pi/4}^{\pi/2} = F(\pi/4) - F(\pi/6) - \left[F(\pi/2) - F(\pi/4)\right]$$

$$= \frac{1}{2}\cdot 1 - \left(\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\right) - \left(0 - \frac{1}{2}\cdot 1\right) = \frac{1}{2} - \frac{\sqrt{3}}{4} + \frac{1}{2} = 1 - \sqrt{3}/4$$

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8) $\frac{1}{|x|}$ è $\frac{1}{x}$ per $x \geq 0$ e $-\frac{1}{x}$ per $x < 0$

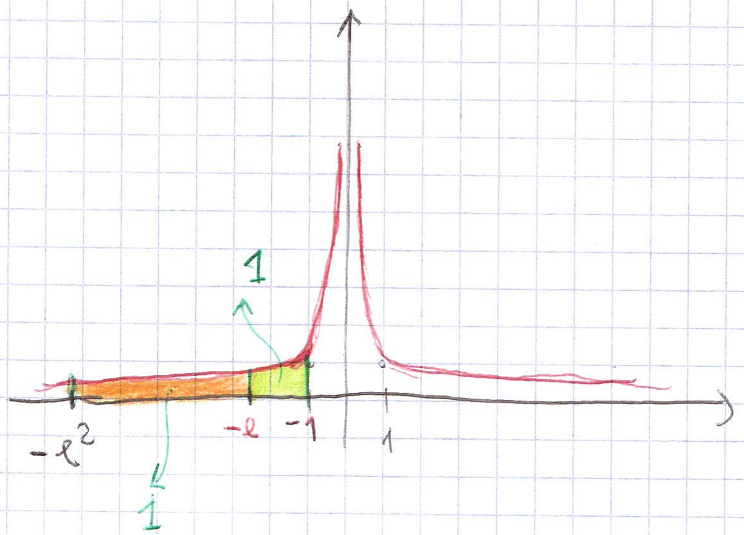
sapendo che $\int \frac{1}{x}dx = \log|x|$ in $[-1,1]$ N.P. $+\infty$

in $[-e,-1]$ si ha $\int_{-e}^{-1} -\frac{1}{x}\,dx = -\int_{-e}^{-1} \frac{1}{x}\,dx = -\log|x|$

$$= \left[-\log|x|\right]_{-e}^{-1} = F(-1) - F(-e) = -\log 1 - (-\log e) = 0 + 1 = \boxed{1}$$

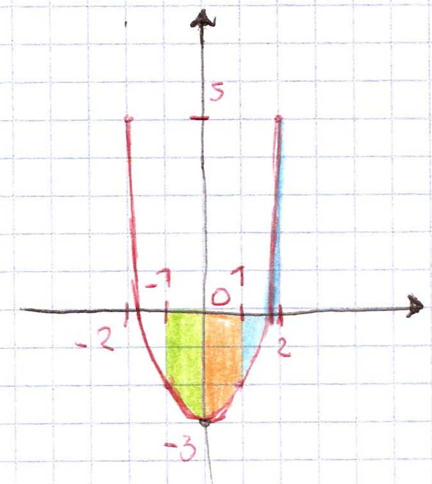
in $[-e^2,-e]$ si ha $\left[-\log|x|\right]_{-e^2}^{-e} = F(-e) - F(-e^2) = -\log e - (-\log e^2)$

$$= -1 + 2 = \boxed{1}$$



9) $|x^3| - 3$ per $x \geq 0$ si ha $x^3 - 3$

per $x < 0$ si ha $-x^3 - 3$



$$\int x^3 - 3 \, dx = \frac{1}{4}x^4 - 3x$$

$$\int -x^3 - 3 \, dx = -\frac{1}{4}x^4 - 3x$$

$[-1, 0] \Rightarrow \left[-\frac{1}{4}x^4 - 3x\right]_{-1}^{0} = F(0) - F(-1) = 0 - \left(-\frac{1}{4} + 3\right) = -\frac{11}{4}$

$[-1, 1]$ dal grafico si vede che è il doppio di $[-1, 0]$

cioè $-\frac{11}{4} \cdot 2 = -\frac{11}{2}$

$[-1, 2]$ dal grafico si vede che è la somma tra $[-1, 1]$ e $[1, 2]$

$[1, 2] \Rightarrow \left[\frac{1}{4}x^4 - 3x\right]_{1}^{2} = F(2) - F(1) = \left(\frac{1}{4} \cdot 16 - 6\right) - \left(\frac{1}{4} \cdot 3\right) = -2 + 11/4$

da cui $[-1, 2] = -2 + \frac{11}{4} - \frac{11}{2} = -\frac{19}{4}$

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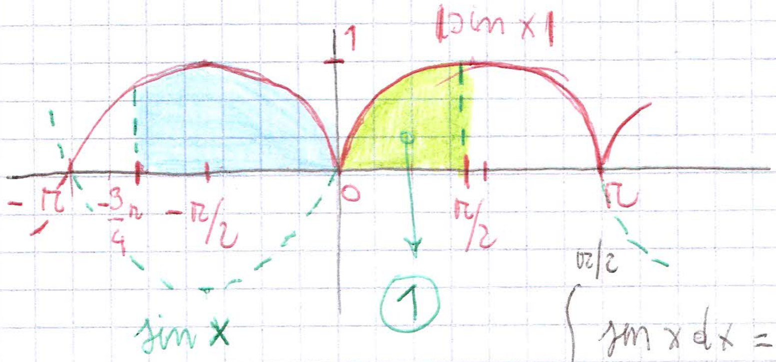
10) $|\sin x| \rightarrow$ è $\sin x$ per $x \in [0, \pi]$

è $-\sin x$ per $x \in [-\pi, 0]$



$$\int \sin x \, dx = -\cos x$$

$$\int_0^{\pi/2} \sin x \, dx = \left[-\cos x\right]_0^{\pi/2} = F(\pi/2) - F(0) = 0 - (-1) = 1$$

essendo noto che $\int_{0}^{\pi/2} \sin x \, dx = 1$, si deduce dal grafico

che in $[-\pi, \pi]$ si ha $\int_{-\pi}^{\pi} |\sin x| \, dx = 4$

e che in $[-\pi, -\pi/2]$ si ha $\int_{-\pi}^{-\pi/2} |\sin x| \, dx = 1$

si può facilmente fare i conti importante è non sbagliare i segni ricordandosi che deve risultare un valore positivo.

$$\int_{-\frac{3}{4}\pi}^{0} |\sin x| \, dx = -\int_{-\frac{3}{4}\pi}^{0} \sin x \, dx = \Big[\cos x \, dx\Big]_{-\frac{3}{4}\pi}^{0} = F(0) - F\left(-\frac{3}{4}\pi\right)$$

$$= 1 - \left(-\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\sqrt{2}}{2} + 1}$$

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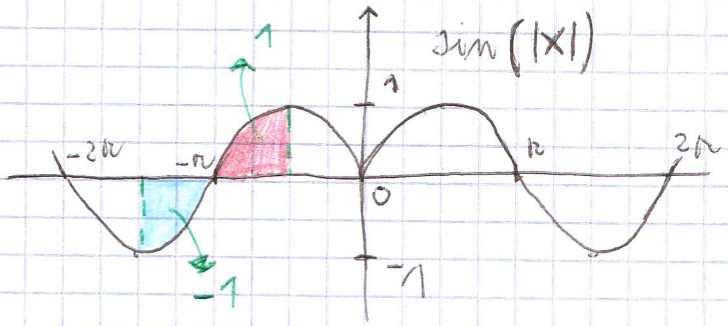
11) $\sin(|x|)$ — per $x \geq 0$ si ha $\sin x$
— per $x < 0$ si ha $\sin(-x) = -\sin(x)$



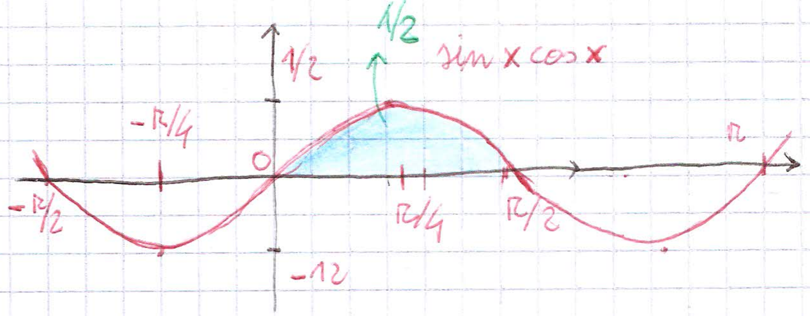
dal grafico e dai risultati dell'esercizio precedente si trovano subito i valori richiesti

in $[-\pi, \pi]$ $\int_{-\pi}^{\pi} \sin(|x|) \, dx = 4$

$[-2\pi, 0]$ $\int_{-2\pi}^{0} \sin(|x|) \, dx = 0 = -\int_{-2\pi}^{0} \sin x \, dx = \Big[\cos x\Big]_{-2\pi}^{0} = F(0) - F(-2\pi) = 1 - 1 = 0$

$[0, \pi]$ $\int_{0}^{\pi} \sin(|x|) \, dx = 2$

12) $\sin x \cos x$



per $x = 0$ e $x = \pi/2$

è uguale a zero

per $x = \frac{\pi}{4}$ si ha $\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{1}{2}$

$\int \sin x \cos x \, dx$ pongo $y = \sin x$ $\frac{dy}{dx} = \cos x$ $dy = \cos x \, dx$

$\int y \, dy = \frac{1}{2} y^2$ torno in x $\frac{1}{2}\sin^2 x$ $\Rightarrow \int \sin x \cos x \, dx = \frac{1}{2}\sin^2 x$

$[0, \frac{\pi}{2}]$ $\left[\frac{1}{2}\sin^2 x\right]_0^{\pi/2} = F(\pi/2) - F(0) = \frac{1}{2} \cdot 1 - 0 = \frac{1}{2}$

$[\pi/2, \pi]$ $\left[\frac{1}{2}\sin^2 x\right]_{\pi/2}^{\pi} = 0 - \left(\frac{1}{2}\right) = -\frac{1}{2}$

$[-\pi/2, 0]$ $\left[\frac{1}{2}\sin^2 x\right]_{-\pi/2}^{0} = 0 - \left(\frac{1}{2} \cdot (-1)^2\right) = -\frac{1}{2}$

risultati evidenti anche senza fare i conti dal grafico

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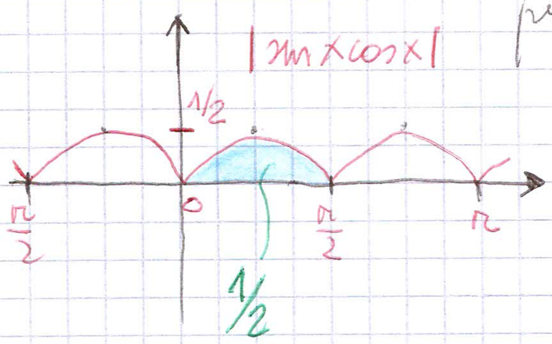
13) $|\sin x \cos x|$ dal grafico e dai risultati dell'esercizio precedente si ricava:



$[0, \pi/2]$ si ha $\int |\sin x \cos x| \, dx = 1/2$

$[\pi/2, \pi]$ si ha $\int \ldots = \frac{1}{2}$

$[-\pi/2, \pi]$ si ha $\int \ldots = 3/2$

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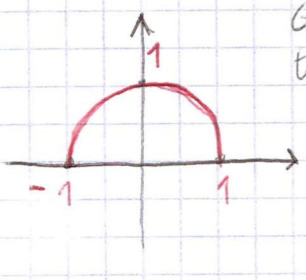
14) $\sqrt{1 - x^2}$ $1 - x^2 \geq 0$ $x \in [-1, 1]$



Geometricamente le soluzioni sono semplici trattandosi di un mezzo cerchio

$[-1, 1] \Rightarrow r^2 \cdot \frac{\pi}{2} = \pi/2$

$[-1, 0] \Rightarrow \frac{\pi}{2} \cdot \frac{1}{2} = \pi/4$

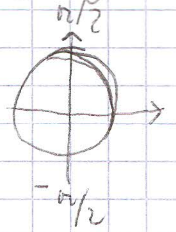
$[0, 2] \Rightarrow$ N.S.

dal punto di vista algebrico mi sembra molto + complicato
ho trovato una sostituzione: $\boxed{\text{meglio vedere es. 19 Lez. 70 - 16-12-10}}$

$\int \sqrt{1-x^2}\, dx$ pongo $x = \sin y$ $\quad y = \arcsin x \quad dx = \cos y\, dy$

$\boxed{x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)}$

$$\int \sqrt{1-\sin^2 y} \cdot \cos y\, dy = \int \sqrt{\cos^2 y}\, \cos y\, dy = \int |\cos y| \cos y\, dy$$



coseno non è negativo

$= \int \cos^2 y\, dy$ con la condizione che $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\cos 2y = \begin{cases} \cos^2 y - \sin^2 y \\ 1 - 2\sin^2 y \\ 2\cos^2 y - 1 \end{cases}$$ Formule equivalenti.

da cui $\cos 2y = 2\cos^2 y - 1 \qquad 2\cos^2 y = \cos 2y + 1 \qquad \cos^2 y = \frac{1+\cos 2y}{2}$ *

oppure

$$\cos 2y = \cos^2 y - \sin^2 y = \cos^2 y - (1-\cos^2 y) = 2\cos^2 y - 1$$

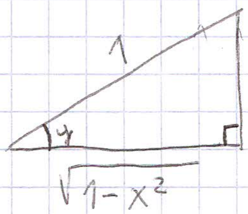
$$\cos 2y = 2\cos^2 y - 1; \qquad \cos^2 y = \frac{1}{2}(1+\cos 2y) = \frac{1}{2} + \frac{1}{2}\cos 2y$$

$$\int \cos^2 y\, dy = \int \left(\frac{1}{2} + \frac{1}{2}\cos 2y\right) dy = \frac{1}{2}y + \frac{1}{2}\sin 2y \cdot \frac{1}{2} = \frac{1}{2}y + \frac{1}{4}\sin 2y$$

essendo
$\sin 2y = 2\sin y \cos y$

da cui $= \frac{1}{2}y + \frac{1}{2}\sin y \cos y$

$\sin y = \frac{x}{1}$



$\cos y = \frac{\sqrt{1-x^2}}{1}$

tornando in x

si ha $\frac{1}{2}\arcsin x + \frac{x\sqrt{1-x^2}}{2}$ $\quad \Big[\quad\Big]_{-1}^{1} = F(1) - F(-1) = \frac{1}{2}\cdot\frac{\pi}{2} - \left(\frac{1}{2}\cdot\left(-\frac{\pi}{2}\right)\right) = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

[Vedere stesso esercizio esempio 6 L. 60 AM1_17]

15) $\cos(2x)\cos(7x)$ sapendo che $\cos\alpha\cos\beta = \frac{1}{2}\left(\cos(\alpha-\beta)+\cos(\alpha+\beta)\right)$

$$\cos(2x)\cdot\cos(7x) = \frac{1}{2}\left(\cos(-5x)+\cos(9x)\right)$$

$$\frac{1}{2}\int\cos(-5x)\,dx + \frac{1}{2}\int\cos 9x\,dx \qquad \int\cos x\,dx = \sin x$$

$$= \frac{1}{2}\cdot\left(-\frac{1}{5}\sin(-5x)\right)+\frac{1}{2}\left(\frac{1}{9}\sin(9x)\right) = -\frac{1}{10}\sin(-5x)+\frac{1}{2}\left(\frac{1}{9}\sin(9x)\right)$$

$$= \frac{1}{10}\sin 5x + \frac{1}{18}\sin 9x\Big]_0^{\pi/2} \quad F(\pi/2)-F(0)$$

$$= \frac{1}{10}\sin\frac{5}{2}\pi + \frac{1}{18}\sin\frac{9\pi}{2} = \frac{1}{10}\sin\pi/2 + \frac{1}{18}\sin\frac{\pi}{2} = \frac{1}{10}+\frac{1}{18} = \frac{14}{90} = 7/45$$

$$= \frac{1}{10}\sin\frac{5}{\ }\cancel{x} + \frac{1}{18}\sin\frac{9}{\ }\cancel{x}\Big]_0^{\pi/4} = F(\pi/4)-F(0) = \frac{1}{10}\sin\frac{5\pi}{4}+\frac{1}{18}\sin\frac{9\pi}{4}$$

$$= \frac{1}{10}\sin\left(-\frac{\pi}{4}\right)+\frac{1}{18}\sin\frac{\pi}{4} = \frac{1}{10}\cdot\left(-\frac{\sqrt{2}}{2}\right)+\frac{1}{18}\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\left(\frac{1}{18}-\frac{1}{10}\right) = -\frac{\cancel{2}}{45}\cdot\frac{\sqrt{2}}{\cancel{2}} = -\sqrt{2}/45$$

$$= \frac{1}{10}\sin\frac{5}{\ }\cancel{x} + \frac{1}{18}\sin\frac{9}{\ }\cancel{x}\Big]_{\pi/6}^{\pi/3} = F(\pi/3)-F(\pi/6) = \frac{1}{180}-\sqrt{3}/20$$

$\boxed{F(\pi/3)}$

$$\frac{1}{10}\sin\frac{5\pi}{3}+\frac{1}{18}\sin\frac{\cancelto{3}{9}\pi}{\cancel{3}} = \frac{1}{10}\sin\left(-\frac{\pi}{3}\right)+\frac{1}{18}\sin 3\pi = \frac{1}{10}\cdot\left(-\frac{\sqrt{3}}{2}\right)+\frac{1}{18}\cdot 0 = \frac{-\sqrt{3}}{20}$$

$$F(\pi/6) = \frac{1}{10}\sin\frac{5\pi}{6}+\frac{1}{18}\sin\frac{\cancelto{3}{9}\pi}{\cancel{6}\,2} = \frac{1}{10}\cdot\sin\frac{\pi}{6}+\frac{1}{18}\sin\frac{3\pi}{2} = \frac{1}{10}\cdot\frac{1}{2}+\frac{1}{18}\cdot(-1)$$

$$= \frac{1}{20}-\frac{1}{18} = \frac{9-10}{180} = -\frac{1}{180} = F(\pi/6)$$

$$F(\pi/3)-F(\pi/6) = \frac{-\sqrt{3}}{20}-\left(-\frac{1}{180}\right) = \frac{1}{180}-\sqrt{3}/20$$

16) $\cos(2x)\sin(7x)$ sapendo che $\cos\alpha\sin\beta =$
$= \frac{1}{2}\left(\sin(\alpha-\beta)+\sin(\alpha+\beta)\right)$

$\cos(2x)\sin 7(x) = \frac{1}{2}\sin(-5x)+\frac{1}{2}\sin(9x)$ attenzione errore applicazione formula di "Werner" o meglio "PRODUCT TO SUM"

$\sin\alpha\cos\beta = \frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right)$ nell esercizio si ha:

$\sin 7x\cos 2x = \frac{1}{2}(\sin 9x + \sin 5x)$

$\int \sin x\, dx = -\cos x$

$\int \frac{1}{2}(\sin 9x + \sin 5x)\, dx = \frac{1}{2}\left(-\frac{1}{9}\cos 9x - \frac{1}{5}\cos 5x\right)$

$= -\frac{1}{10}\cos 5x - \frac{1}{18}\cos 9x$

$\Big]_0^{\pi/2} F(\pi/2)-F(0) = 0 - \left(-\frac{1}{10}\cos 0 - \frac{1}{18}\cos 0\right) = -\left(-\frac{1}{10}-\frac{1}{18}\right) = 7/45$

$\Big]_0^{\pi/4} = F(\pi/4) - F(0) = -\frac{1}{10}\cos\frac{5\pi}{4} - \frac{1}{18}\cos\frac{9\pi}{4} = -\frac{1}{10}\left(-\cos\frac{\pi}{4}\right) - \frac{1}{18}\cos\frac{\pi}{4}$

$7/45$

$= -\frac{1}{10}\left(-\frac{\sqrt{2}}{2}\right) - \frac{1}{18}\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{20} - \frac{\sqrt{2}}{36} = \frac{4\sqrt{2}}{180} = \frac{\sqrt{2}}{45}$ da cui $\Big]_0^{\pi/4} = \frac{\sqrt{2}}{45} + \frac{7}{45}$

$\Big]_{\pi/6}^{\pi/3} = F(\pi/3) - F(\pi/6) = \frac{1}{180} - \sqrt{3}/20$

$F(\pi/3) = -\frac{1}{10}\cos\frac{5\pi}{3} - \frac{1}{18}\cos\frac{9\pi}{3} = -\frac{1}{10}\cos\frac{\pi}{3} - \frac{1}{18}\cos 3\pi$

$= -\frac{1}{10}\left(\frac{1}{2}\right) - \frac{1}{18}(-1) = -\frac{1}{20} + \frac{1}{18} = \frac{1}{180}$

$F(\pi/6) = -\left(-\frac{1}{10}\cos\frac{5\pi}{6} - \frac{1}{18}\cos\frac{9\pi}{6}\right) = -\left(-\frac{1}{10}\left(-\cos\frac{\pi}{6}\right) - \frac{1}{18}\cos\frac{3\pi}{2}\right)$ 0

$= -\left(-\frac{1}{10}\cdot\left(\frac{-\sqrt{3}}{2}\right)\right) = -\sqrt{3}/20$