RIEMANN SURFACES AND ALGEBRAIC CURVES - GEOMETRIA ALGEBRICA C

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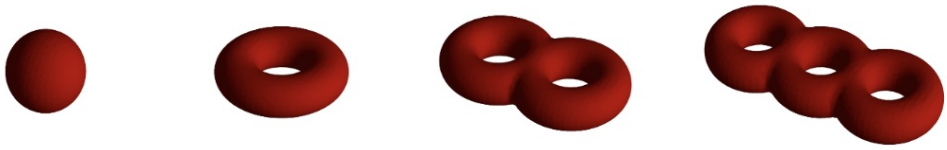
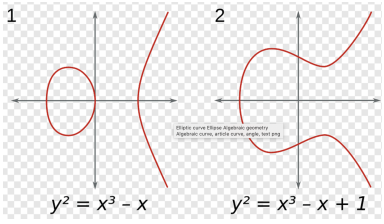
Teacher: Marco Franciosi
marco.franciosi@unipi.it

Introduction

A *Riemann Surface* is a connected one-dimensional complex manifold. From a topological point of view, a compact Riemann surface is just an orientable two-dimensional topological manifold. In the compact case it is classified by its genus.

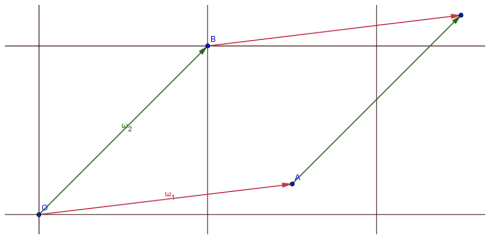


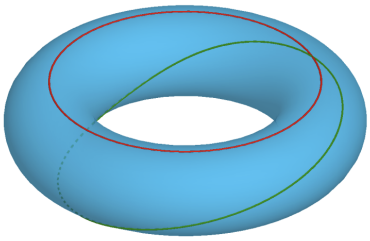
To study Riemann surfaces we use complex charts and meromorphic function. A key point is the notion of holomorphic map between Riemann surfaces.



On the other side there is the notion of *Algebraic Curve*. It is one of the oldest subjects in modern mathematics, as it was one of the first things people did once they learned about polynomials. An algebraic curve is a one-dimensional complex variety (affine or projective) defined via the zero-locus of polynomials. The simplest case is given by affine plane curves $\{(x, y) \in \mathbb{C}^2 \ : \ p(x, y) = 0\}$, where p is a polynomial.

The aim of the course is to describe Riemann Surfaces, Algebraic Curves, and to describe the bridge between these two categories.





Topics

We will start with the basic properties of Riemann surface, we will consider meromorphic function and we will study holomorphic function between two Riemann surfaces. Then we will introduce the notion of differential forms and we'll see their behavior.

Concerning the notion of Algebraic curves we will introduce the language of modern algebraic geometry: sheaves, divisors, bundles, cohomology. We will see the famous "Riemann-Roch Theorem" and the impact of "Serre duality". Then we will show how any compact Riemann surface carries a natural structure of an algebraic curve and we will conclude by studying the Jacobian associated to a Riemann Surface.

Practical information
The course will last 42 hours, and it will take place in the second semester. There will be an oral examination, including a short seminar and questions about the content of the course. The prerequisites for this course are basic complex analysis, the first notions of Algebraic Topology and Algebraic Geometry.

Website
`http://pagine.dm.unipi.it/∼a008702/geometria-algebrica-c.html`

Literature

[1] R. Miranda, *Algebraic curves and Riemann surfaces*, Graduate Studies in Mathematics, Vol. 5, American Mathematical Society.

[2] F. Kirwan, *Complex algebraic curves*, London Mathematical Society, Student texts 23.

[3] E. Arbarello, M. Cornalba, P.A Griffiths, J. Harris *Geometry of algebraic curves, Vol. I.* Grundlehren der Mathematischen Wissenschaften, 267. Springer-Verlag